

Mark Scheme (Results) January 2009

GCE

GCE Mathematics (6664/01)





January 2009 6664 Core Mathematics C2 Mark Scheme

Question Number	Scheme	Marks	
1	$(3-2x)^5 = 243$, $+5 \times (3)^4 (-2x) = -810x$	B1, B1	
	$+\frac{5\times4}{2}(3)^3(-2x)^2 = +1080x^2$	M1 A1	(4)
			[4]
Notes	First term must be 243 for B1 , writing just 3^5 is B0 (Mark their final answe second line of special cases below). Term must be simplified to $-810x$ for B1 The <i>x</i> is required for this mark. The method mark (M1) is generous and is awarded for an attempt at Binor	rs except in	ıe
	third term.	indi to get ti	
	There must be an x^2 (or no x- i.e. not wrong power) and attempt at Binomia and at dealing with powers of 3 and 2. The power of 3 should not be one, bu 2 may be one (regarded as bracketing slip).	al Coefficier at the power	nt of
	So allow $\begin{pmatrix} 5\\2 \end{pmatrix}$ or $\begin{pmatrix} 5\\3 \end{pmatrix}$ or ${}^{5}C_{2}$ or ${}^{5}C_{3}$ or even $\begin{pmatrix} 5\\2 \end{pmatrix}$ or $\begin{pmatrix} 5\\3 \end{pmatrix}$ or use of '10' (m	aybe from	
	Pascal's triangle)		
	May see ${}^{5}C_{2}(3)^{3}(-2x)^{2}$ or ${}^{5}C_{2}(3)^{3}(-2x^{2})$ or ${}^{5}C_{2}(3)^{5}(-\frac{2}{3}x^{2})$ or $10(3)^{3}(2x)^{2}x^{2}$	which would	l
	each score the M1 A lie a a and needs $1080r^2$ (if $1080r^2$ is written with no working this is a	wardad batk	
	marks i.e. M1 A1.)		1
Special	$243+810x+1080x^2$ is B1B0M1A1 (condone no negative signs)		
cases	Follows correct answer with $27-90x+120x^2$ can isw here (sp case)– full a correct answer	marks for	
	Misreads <i>ascending</i> and gives $-32x^5 + 240x^4 - 720x^3$ is marked as B1B0M case and must be completely correct. (<i>If any slips could get B0B0M1A0</i>) Ignores 3 and expands $(1+2x)^5$ is 0 /4	[1A0 special	l
	243 -810x $1080x^2$ is full marks but 243 -810 1080 is B1.B0.M1.A0		
	NB Alternative method $3^5(1-\frac{2}{3}x)^5 = 3^5 - 5 \times 3^5 \times (\frac{2}{3}x) + (\frac{5}{3})3^5(-\frac{2}{3}x)^2 +$ is	B0B0M1A	0
	- answers must be simplified to $243 - 810x + 1080x^2$ for full marks (awarded	d as before)	
	Special case $3(1-\frac{2}{3}x)^5 = 3-5 \times 3 \times (\frac{2}{3}x) + {\binom{5}{3}} 3(-\frac{2}{3}x)^2 +$ is B0 , B0 , M1 , A	.0	
	Or $3(1-2x)^5$ is B0B0M0A0		

Question Number	Scheme	Marks
2	$y = (1 + x)(4 - x) = 4 + 3x - x^2$ M: Expand, giving 3 (or 4) terms	M1
	$\int (4+3x-x^2) dx = 4x + \frac{3x^2}{2} - \frac{x^3}{3}$ M: Attempt to integrate	M1 A1
	$= \left[\dots \right]_{-1}^{4} = \left(16 + 24 - \frac{64}{3} \right) - \left(-4 + \frac{3}{2} + \frac{1}{3} \right) = \frac{125}{6} \qquad \left(= 20\frac{5}{6} \right)$	M1 A1 (5) [5]
Notes	M1 needs expansion, there may be a slip involving a sign or simple arithme $1 \times 4 = 5$, but there needs to be a 'constant' an 'r term' and an 'r' term'. The	etical error e.g.
	not need to be collected. (Need not be seen if next line correct)	
	Attempt to integrate means that $x^n \rightarrow x^{n+1}$ for at least one of the terms, then awarded (even 4 becoming $4x$ is sufficient) – one correct power sufficient.	n M1 is
	A1 is for correct answer only, not follow through. But allow $2x^2 - \frac{1}{2}x^2$ or an	ny correct
	equivalent. Allow $+ c$, and even allow an evaluated extra constant term.	
	M1 : Substitute limit 4 and limit -1 into a changed function (must be -1) and subtraction (either way round).	d indicate
	A1 must be exact, not 20.83 or similar. If recurring indicated can have the model Negative area, even if subsequently positive loses the A mark.	nark.
Special cases	 (i) Uses calculator method: M1 for expansion (if seen) M1 for limits if answ 0, 1 or 2 marks out of 5 is possible (Most likely M0 M0 A0 M1 A0) (ii) Uses trapezium rule : not exact, no calculus – 0/5 unless expansion mark (iii) Using original method, but then change all signs after expansion is like M1 M1 A0, M1 A0 i.e. 3/5 	wer correct, so k M1 gained. ly to lead to:

Question Number	Scheme	Marks
3 (a)	3.84, 4.14, 4.58 (Any one correct B1 B0. All correct B1 B1)	B1 B1 (2)
(b)	$\frac{1}{2} \times 0.4$, {(3+4.58)+2(3.47+3.84+4.14+4.39)}	B1, M1 A1ft
	= 7.852 (awrt 7.9)	A1 (4)
		[6]
Notes (a)	B1 for one answer correct Second B1 for all three correct	
	Accept awrt ones given or exact answers so $\sqrt{21}$, $\sqrt{\left(\frac{369}{25}\right)}$ or $\frac{3\sqrt{41}}{5}$, and	$\sqrt{\left(\frac{429}{25}\right)}$ or
	$\frac{\sqrt{429}}{5}$, score the marks.	
(b)	B1 is for using 0.2 or $\frac{0.4}{2}$ as $\frac{1}{2}h$.	
	M1 requires first bracket to contain first plus last values and second bracket to include no additional values from those in the table. If the only mistake is to omit one value from 2^{nd} bracket this may be rega can be allowed (An extra repeated term forfeits the M mark however) <i>x</i> values: M0 if values used in brackets are <i>x</i> values instead of <i>y</i> values. Separate trapezia may be used : B1 for 0.2, M1 for $\frac{1}{2}h(a+b)$ used 4 or 5 tim	urded as a slip an nes (and A1 ft all
	e.g. $0.2(3+3.47) + 0.2(3.47+3.84) + 0.2(3.84+4.14) + 0.2(4.14+4.58)$ is N	M1 A0
	Alft follows their answers to part (a) and is for {correct expression}	
	Final A1 must be correct. (No follow through)	
Special cases	Bracketing mistake: i.e. $\frac{1}{2} \times 0.4(3+4.58) + 2(3.47+3.84+4.14+4.39)$	
	scores B1 M1 A0 A0 <u>unless</u> the final answer implies that the calculation has been done correctly (then full marks can be given).	
	Need to see trapezium rule – answer only (with no working) is 0/4.	

Question Number	Scheme	Marks
4	$2\log_5 x = \log_5(x^2), \qquad \log_5(4-x) - \log_5(x^2) = \log_5\frac{4-x}{x^2}$	B1, M1
	$\log\left(\frac{4-x}{x^2}\right) = \log 5$ $5x^2 + x - 4 = 0$ or $5x^2 + x = 4$ o.e.	M1 A1
	$(5x-4)(x+1) = 0$ $x = \frac{4}{5}$ $(x = -1)$	dM1 A1 (6) [6]
Notes	B1 is awarded for $2\log x = \log x^2$ anywhere. M1 for correct use of $\log A - \log B = \log \frac{A}{B}$ M1 for replacing 1 by $\log_k k$. A1 for correct quadratic $(\log(4-x) - \log x^2 = \log 5 \Rightarrow 4 - x - x^2 = 5$ is B1M0M1A0 M0A0) dM1 for attempt to solve quadratic with usual conventions. (Only award if M marks have been awarded) A1 for 4/5 or 0.8 or equivalent (Ignore extra answer).	f previous two
Alternative 1	$\log_{5}(4-x) - 1 = 2\log_{5} x \text{ so } \log_{5}(4-x) - \log_{5} 5 = 2\log_{5} x$ $\log_{5} \frac{4-x}{5} = 2\log_{5} x$ then could complete solution with $2\log_{5} x = \log_{5}(x^{2})$ $\left(\frac{4-x}{5}\right) = x^{2} \qquad 5x^{2} + x - 4 = 0$ Then as in first method $(5x-4)(x+1) = 0 \qquad x = \frac{4}{5} \qquad (x = -1)$	M1 M1 B1 A1 dM1 A1 (6) [6]
Special cases	Complete trial and error yielding 0.8 is M3 and B1 for 0.8 A1, A1 awarded for each of two tries evaluated. i.e. 6/6 Incomplete trial and error with wrong or no solution is 0/6 Just answer 0.8 with no working is B1 If log base 10 or base e used throughout - can score B1M1M1A0M1A0	

Question Number	Scheme	Marks
5 (a)	<i>PQ</i> : $m_1 = \frac{10-2}{9-(-3)}$ (= $\frac{2}{3}$) and <i>QR</i> : $m_2 = \frac{10-4}{9-a}$	M1
(b) Alt for (a)	$m_1m_2 = -1: \frac{8}{12} \times \frac{6}{9-a} = -1 \qquad a = 13 \qquad (*)$ (a) Alternative method (Pythagoras) Finds all three of the following	M1 A1 (3)
(u)	$(9-(-3))^2 + (10-2)^2$, (<i>i.e.</i> 208), $(9-a)^2 + (10-4)^2$, $(a-(-3))^2 + (4-2)^2$ Using Pythagoras (correct way around) e.g. $a^2 + 6a + 9 = 240 + a^2 - 18a + 81$ to form equation Solve (or verify) for <i>a</i> , <i>a</i> = 13 (*) (b) Centre is at (5, 3)	M1 M1 A1 B1
	$(r^2 =) (10-3)^2 + (9-5)^2$ or equiv., or $(d^2 =) (13-(-3))^2 + (4-2)^2$ $(x-5)^2 + (y-3)^2 = 65$ or $x^2 + y^2 - 10x - 6y - 31 = 0$	M1 A1 M1 A1 (5)
Alt for (b)	Uses $(x-a)^2 + (y-b)^2 = r^2$ or $x^2 + y^2 + 2gx + 2fy + c = 0$ and substitutes (-3, 2), (9, 10) and (13, 4) then eliminates one unknown Eliminates second unknown	M1 M1
Netze	Obtains $g = -5$, $f = -3$, $c = -31$ or $a = 5$, $b = 3$, $r^2 = 65$	A1, A1, B1cao (5) [8]
Notes (a)	M1-considers gradients of PQ and QR -must be y difference / x difference (or considers three lengths as in alternative method) M1 Substitutes gradients into product = -1 (or lengths into Pythagoras' Theorem correct way round) A1 Obtains $a = 13$ with no errors by solution or verification. Verification can sco	the re 3/3.
(b)	Geometrical method: B1 for coordinates of centre – can be implied by use in par	t (b)
	M1 for attempt to find r^2 , d^2 , r or d (allow one slip in a bracket).	
	A1 cao. These two marks may be gained implicitly from circle equation	
	M1 for $(x \pm 5)^2 + (y \pm 3)^2 = k^2$ or $(x \pm 3)^2 + (y \pm 5)^2 = k^2$ ft their (5,3) Allow k^2 n numerical.	on
	A1 cao for whole equation and rhs must be 65 or $(\sqrt{65})^2$, (similarly B1 must be	65 or
	$\left(\sqrt{65}\right)^2$, in alternative method for (b))	

Question Number	Scheme	Marks
Further alternatives	(i) A number of methods find gradient of PQ = 2/3 then give perpendicular gradient is $-3/2$ This is M1 They then proceed using equations of lines through point Q or by using gradient QR to obtain equation such as $\frac{4-10}{a-9} = -\frac{3}{2}$ M1 (may still have x in this equation rather than a and there may be a small slip) They then complete to give (a) = 13 A1	M1 M1 A1
	(ii) A long involved method has been seen finding the coordinates of the centre of the circle first. This can be done by a variety of methods Giving centre as (c, 3) and using an equation such as $(c-9)^2 + 7^2 = (c+3)^2 + 1^2$ (equal radii) or $\frac{3-6}{c-3} = -\frac{3}{2}$ M1 (perpendicular from centre to chord bisects chord)	M1
	Then using $c (= 5)$ to find a is M1	M1
	Finally $a = 13$ A1	A1
	(iii) Vector Method: States PO OP = 0 with vectors stated 12i \pm 8i and (0 = c) i \pm 6i is M1	M1
	Evaluates scalar product so $108 - 12a + 48 = 0$ (M1)	M1
	solves to give $a = 13$ (A1)	A1

Question Number	Scheme	Marks	
6 (a)	f(2) = 16 + 40 + 2a + b or $f(-1) = 1 - 5 - a + b$	M1 A1	
	Finds 2nd remainder and equates to $1st \Rightarrow 16+40+2a+b=1-5-a+b$	M1 A1	
(b)	a = -20 f(-3) = (-3) ⁴ + 5(-3) ³ - 3a + b = 0	A1cso (M1 A1ft	(5)
	81 - 135 + 60 + b = 0 gives $b = -6$	A1 cso	(3) [8]
Alternative for (a)	(a) Uses long division, to get remainders as $b + 2a + 56$ or $b - a - 4$ or correct equivalent	M1 A1	
	Uses second long division as far as remainder term, to get b + 2a + 56 = b - a - 4 or correct equivalent a = -20	M1 A1 A1cso	(5)
Alternative for (b)	(b) Uses long division of $x^4 + 5x^3 - 20x + b$ by $(x + 3)$ to obtain $x^3 + 2x^2 - 6x + a + 18$ (with their value for a)	M1 A1ft	. ,
	Giving remainder $b + 6 = 0$ and so $b = -6$	A1 cso	(3) [8]
Notes (a)	M1 : Attempts $f(\pm 2)$ or $f(\pm 1)$ A1 is for the answer shown (or simplified with terms collected) for on M1: Attempts other remainder and puts one equal to the other A1: for correct equation in <i>a</i> (and <i>b</i>) then A1 for $a = -20$ cso	e remainde	er
(b)	M1 : Puts $f(\pm 3) = 0$		
	A1 is for $f(-3) = 0$, (where f is original function), with no sign or subst (follow through on 'a' and could still be in terms of a) A1: $b = -6$ is cso.	titution erro	ors
Alternatives	 (a) M1: Uses long division of x⁴ + 5x³ + ax + b by (x ±2) or by (x ±1) as term quotient A1: Obtains at least one correct remainder M1: Obtains second remainder and puts two remainders (no x terms) e A1: correct equation A1: correct answer a = -20 following correct w (b) M1: complete long division as far as constant (ignore remainder) 	far as three qual ork.	e
	A1ft: needs correct answer for their <i>a</i> A1: correct answer		
Beware: It is	possible to get correct answers with wrong working. If remainders are ec	juated to 0	in
part (a) both	correct answers are obtained fortuitously. This could score M1A1M0A0A01	M1A1A0	

Question Number		Scheme	Marks	5
7	(a)	$\frac{1}{2}r^2\theta = \frac{1}{2} \times 6^2 \times 2.2 = 39.6$ (cm ²)	M1 A1	(2)
	(b)	$\left(\frac{2\pi - 2.2}{2}\right) = \pi - 1.1 = 2.04$ (rad)	M1 A1	(2)
		(c) $\Delta DAC = \frac{1}{2} \times 6 \times 4 \sin 2.04$ (\$\approx 10.7)	M1 A1ft	
		Total area = sector + 2 triangles = 61 (cm^2)	M1 A1	(4) [8]
	(a)	M1: Needs θ in radians for this formula. Could convert to degrees and use degrees formula.		
		A1: Does not need units. Answer should be 39.6 exactly. Answer with no working is M1 A1. This M1A1 can only be awarded in part (a).		
	(b)	M1: Needs full method to give angle in radians A1: Allow answers which round to 2.04 (Just writes 2.04 – no working i	s 2/2)	
	(c)	M1: Use $\frac{1}{2} \times 6 \times 4 \sin A$ (if any other triangle formula e.g. $\frac{1}{2}b \times h$ is use	d the meth	od
		 must be complete for this mark) (No value needed for A, but should not I A1: ft the value obtained in part (b) – need not be evaluated- could be in M1: Uses Total area = sector + 2 triangles or other complete method A1: Allow answers which round to 61. (Do not need units) 	be using 2.2 n degrees	2)
		Special case degrees: Could get M0A0, M0A0, M1A1M1A0 Special case: Use $\Delta BDC - \Delta BAC$ Both areas needed for first M1 Total area = sector + area found is second M1 NB Just finding lengths BD, DC, and angle BDC then assuming area BDC is find area BDC is $0/4$	is a sector t	0

Question Number	Scheme	Marks	
8 (a)		M1 A1	(2)
(u) (b)	$4(1 - \cos^{2} x) + 9\cos x - 6 = 0 \qquad 4\cos^{2} x - 9\cos x + 2 = 0 (*)$	M1 A1	(2)
	$(4\cos x - 1)(\cos x - 2) = 0$ $\cos x =, \frac{1}{4}$	B1	
	$360 - \alpha$, $360 + \alpha$ or $720 - \alpha$	M1, M1	
	284.5, 435.5, 644.5	A1	(6) [8]
(a)	M1: Uses $\sin^2 x = 1 - \cos^2 x$ (may omit bracket) not $\sin^2 x = \cos^2 x - 1$ A1: Obtains the printed answer without error – must have = 0		
(b)	M1: Solves the quadratic with usual conventions A1: Obtains ¹ / ₄ accurately- ignore extra answer 2 but penalise e.g2. B1: allow answers which round to 75.5 M1: $360 - \alpha$ ft their value, M1: $360 + \alpha$ ft their value or 720 - α ft	·	
Special cases	A1: Three and only three correct exact answers in the range achieves the In part (b) Error in solving quadratic (4cosx-1)(cosx+2) Could yield, M1A0B1M1M1A1 losing one mark for the error	ie mark	
	Works in radians: Complete work in radians :Obtains 1.3 B0 . Then allow M1 M1 for $2\pi - \alpha$ $4\pi - \alpha$ Then gets 5.0, 7.6, 11.3 A0 so 2/4	α , $2\pi + \alpha$ of)r
	Mixed answer 1.3, 360 – 1.3, 360 + 1.3, 720 – 1.3 still gets B0M1M1A0		

Question Number	Scheme	Mark	(S
9 (a)	Initial step: Two of: $a = k + 4$, $ar = k$, $ar^2 = 2k - 15$ Or one of: $r = \frac{k}{k+4}$, $r = \frac{2k - 15}{k}$, $r^2 = \frac{2k - 15}{k+4}$, Or $k = \sqrt{(k+4)(2k-15)}$ or even $k^3 = (k+4)k(2k-15)$ $k^2 = (k+4)(2k-15)$, so $k^2 = 2k^2 + 8k - 15k - 60$ Proceed to $k^2 - 7k - 60 = 0$ (*)	M1 M1, A1 A1	(4)
(b)	(k-12)(k+5) = 0 $k = 12$ (*)	M1 A1	(2)
(c)	Common ratio: $\frac{k}{k+4}$ or $\frac{2k-15}{k} = \frac{12}{16} \left(= \frac{3}{4} \text{ or } 0.75 \right)$	M1 A1	(2)
(d)	$\frac{a}{1-r} = \frac{16}{\binom{1}{4}} = 64$	M1 A1	(2) [10]
(a) (b) (c) (d)	M1: The 'initial step', scoring the first M mark, may be implied by next lin M1: Eliminates <i>a</i> and <i>r</i> to give valid equation in <i>k</i> only. Can be awarded for involving fractions. A1 : need some correct expansion and working and answer equivalent to re quadratic but with uncollected terms. Equations involving fractions do not g (No fractions, no brackets – could be a cubic equation) A1: as answer is printed this mark is for cso (Needs = 0) All four marks must be scored in part (a) M1: Attempt to solve quadratic A1: This is for correct factorisation or solution and $k = 12$. Ignore the extra -5 or even $k = 5$), if seen. Substitute and verify is M1 A0 Marks must be scored in part (b) M1: Complete method to find <i>r</i> Could have answer in terms of <i>k</i> A1: 0.75 or any correct equivalent Both Marks must be scored in (c) M1: Tries to use $\frac{a}{1-r}$, (even with $r > 1$). Could have an answer still in terms A1: This answer is 64 cao.	e of proof r equation quired get this ma solution (<i>i</i>	rk. k =

Question Number	Scheme	Marks
10 (a)	$2\pi rh + 2\pi r^{2} = 800$ $h = \frac{400 - \pi r^{2}}{V} = \pi r^{2} \left(\frac{400 - \pi r^{2}}{V}\right) = 400r - \pi r^{3} \qquad (*)$	B1 M1, M1 A1 (4)
(b)	$\frac{dV}{dr} = 400 - 3\pi r^2$	M1 A1
	$400 - 3\pi r^2 = 0$ $r^2 =,$ $r = \sqrt{\frac{400}{3\pi}}$ (= 6.5 (2 s.f.))	M1 A1
	$V = 400r - \pi r^{3} = 1737 = \frac{800}{3} \sqrt{\frac{400}{3\pi}} \text{ (cm}^{3}\text{)}$	M1 A1 (6)
(c)	$\frac{d^2 V}{dr^2} = -6\pi r$, Negative, \therefore maximum (Parts (b) and (c) should be considered together when marking)	M1 A1 (2) [12]
<u>Other</u> <u>methods</u> <u>for part</u> (c):	<u>Either:</u> M: Find <u>value</u> of $\frac{dV}{dr}$ on each side of " $r = \sqrt{\frac{400}{3\pi}}$ " and consider sign.	<u> </u>
	A: Indicate sign change of positive to negative for $\frac{dv}{dr}$, and conclude max.	
	<u>Or:</u> M: Find <u>value</u> of V on each side of " $r = \sqrt{\frac{400}{3\pi}}$ " and compare with "1737"	" ·
	A: Indicate that both values are less than 1737 or 1737.25, and conclude max	Κ.
Notes (a)	B1: For any correct form of this equation (may be unsimplified, may be i M1)	mplied by 1 st
	M1 : Making <i>h</i> the subject of their three or four term formula M1: Substituting expression for <i>h</i> into $\pi r^2 h$ (independent mark) Must n expression in <i>r</i> only. A1: cso	ow be
(b)	M1: At least one power of <i>r</i> decreased by 1 A1: cao dV	
	M1: Setting $\frac{dr}{dr} = 0$ and finding a value for correct power of <i>r</i> for candida	te
	A1: This mark may be credited if the value of V is correct. Otherwise ans round to 6.5 (allow ± 6.5) or be exact answer.	wers should
	M1: Substitute a positive value of r to give V A1: 1737 or 1737.25 of answer	or exact

(c)	M1: needs complete method e.g.attempts differentiation (power reduced) of their first derivative and
	considers its sign
	A1(first method) should be $-6\pi r$ (do not need to substitute r and can condone wrong
	<i>r</i> if found in (b))
	Need to conclude maximum or indicate by a tick that it is maximum. Throughout allow confused notation such as dy/dx for dV/dr
Alternative for (a)	$A = 2\pi r^2 + 2\pi rh$, $\frac{A}{2} \times r = \pi r^3 + \pi r^2 h$ is M1 Equate to 400 <i>r</i> B1
	Then $V = 400r - \pi r^3$ is M1 A1

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