

Mark Scheme (Results)

Summer 2015

Pearson Edexcel GCE in  
Core Mathematics 4 (6666/01)

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Summer 2015

Publications Code UA041202

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

## EDEXCEL GCE MATHEMATICS

### General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
  - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
  - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
  - **B** marks are unconditional accuracy marks (independent of M marks)
  - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
  - ft – follow through
  - the symbol  $\surd$  will be used for correct ft
  - cao – correct answer only
  - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
  - isw – ignore subsequent working
  - awrt – answers which round to
  - SC: special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - dp decimal places
  - sf significant figures
  - \* The answer is printed on the paper
  - $\square$  The second mark is dependent on gaining the first mark
  - dM1 denotes a method mark which is dependent upon the award of the previous method mark.
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
  5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
  - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
  - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
7. Ignore wrong working or incorrect statements following a correct answer.

## General Principles for Core Mathematics Marking

*(But note that specific mark schemes may sometimes override these general principles).*

### **Method mark for solving 3 term quadratic:**

#### **1. Factorisation**

$(x^2 + bx + c) = (x + p)(x + q)$ , where  $|pq| = |c|$ , leading to  $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$ , where  $|pq| = |c|$  and  $|mn| = |a|$ , leading to  $x = \dots$

#### **2. Formula**

Attempt to use the correct formula (with values for a, b and c).

#### **3. Completing the square**

Solving  $x^2 + bx + c = 0$ :  $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$ ,  $q \neq 0$ , leading to  $x = \dots$

### **Method marks for differentiation and integration:**

#### **1. Differentiation**

Power of at least one term decreased by 1. ( $x^n \rightarrow x^{n-1}$ )

#### **2. Integration**

Power of at least one term increased by 1. ( $x^n \rightarrow x^{n+1}$ )

### **Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

### **Exact answers**

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.



June 2015  
6666/01 Core Mathematics 4  
Mark Scheme

Question Number	Scheme	Marks	
<b>1. (a)</b>	$(4 + 5x)^{\frac{1}{2}} = \underline{(4)}^{\frac{1}{2}} \left( 1 + \frac{5x}{4} \right)^{\frac{1}{2}} = \underline{2} \left( 1 + \frac{5x}{4} \right)^{\frac{1}{2}} \qquad \underline{(4)}^{\frac{1}{2}} \text{ or } \underline{2}$	<b>B1</b>	
	$= \{2\} \left[ 1 + \left( \frac{1}{2} \right) (kx) + \frac{\left( \frac{1}{2} \right) \left( -\frac{1}{2} \right)}{2!} (kx)^2 + \dots \right]$	see notes	M1 A1ft
	$= \{2\} \left[ 1 + \left( \frac{1}{2} \right) \left( \frac{5x}{4} \right) + \frac{\left( \frac{1}{2} \right) \left( -\frac{1}{2} \right)}{2!} \left( \frac{5x}{4} \right)^2 + \dots \right]$		
	$= 2 \left[ 1 + \frac{5}{8}x - \frac{25}{128}x^2 + \dots \right]$	See notes below!	
	$= 2 + \frac{5}{4}x; - \frac{25}{64}x^2 + \dots$	isw	A1; A1
		<b>[5]</b>	
<b>(b)</b>	$\left\{ x = \frac{1}{10} \Rightarrow (4 + 5(0.1))^{\frac{1}{2}} = \sqrt{4.5} = \sqrt{\frac{9}{2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{\sqrt{2}\sqrt{2}} \right\}$		
	$= \frac{3}{2}\sqrt{2}$	$\frac{3}{2}\sqrt{2}$ or $k = \frac{3}{2}$ or 1.5 o.e.	B1
			<b>[1]</b>
<b>(c)</b>	$\frac{3}{2}\sqrt{2} \text{ or } 1.5\sqrt{2} \text{ or } \frac{3}{\sqrt{2}} = 2 + \frac{5}{4}\left(\frac{1}{10}\right) - \frac{25}{64}\left(\frac{1}{10}\right)^2 + \dots \{= 2.121\dots\}$	See notes	M1
	$\text{So, } \frac{3}{2}\sqrt{2} = \frac{543}{256} \text{ or } \frac{3}{\sqrt{2}} = \frac{543}{256}$		
	$\text{yields, } \sqrt{2} = \frac{181}{128} \text{ or } \sqrt{2} = \frac{256}{181}$	$\frac{181}{128}$ or $\frac{362}{256}$ or $\frac{543}{384}$ or $\frac{256}{181}$ etc.	A1 oe
			<b>[2]</b>
<b>Question 1 Notes</b>			<b>8</b>
<b>1. (a)</b>	<b>B1</b>	$\underline{(4)}^{\frac{1}{2}}$ or $\underline{2}$ outside brackets or $\underline{2}$ as candidate's constant term in their binomial expansion.	
	<b>M1</b>	Expands $(\dots + kx)^{\frac{1}{2}}$ to give any 2 terms out of 3 terms simplified or un-simplified, Eg: $1 + \left(\frac{1}{2}\right)(kx)$ or $\left(\frac{1}{2}\right)(kx) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(kx)^2$ or $1 + \dots + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(kx)^2$ where $k$ is a numerical value and <b>where</b> $k \neq 1$ .	
	<b>A1</b>	A correct simplified or un-simplified $1 + \left(\frac{1}{2}\right)(kx) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(kx)^2$ expansion with <b>consistent</b> $(kx)$ .	
<b>Note</b>	$(kx)$ , $k \neq 1$ , must be consistent (on the RHS, not necessarily on the LHS) in a candidate's expansion.		

1. (a) ctd.	<b>Note</b>	Award B1M1A0 for $2 \left[ 1 + \left(\frac{1}{2}\right)(5x) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!} \left(\frac{5x}{4}\right)^2 + \dots \right]$ because $(kx)$ is not consistent.
	<b>Note</b>	<b>Incorrect bracketing:</b> $2 \left[ 1 + \left(\frac{1}{2}\right)\left(\frac{5x}{4}\right) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!} \left(\frac{5x^2}{4}\right) + \dots \right]$ is B1M1A0 unless recovered.
	<b>A1</b>	$2 + \frac{5}{4}x$ ( <b>simplified fractions</b> ) or allow $2 + 1.25x$ or $2 + 1\frac{1}{4}x$
	<b>A1</b>	Accept only $-\frac{25}{64}x^2$ or $-0.390625x^2$
	<b>SC</b>	If a candidate <i>would otherwise score</i> 2 <sup>nd</sup> A0, 3 <sup>rd</sup> A0 then <b>allow Special Case 2<sup>nd</sup> A1 for either</b> <b>SC:</b> $2 \left[ 1 + \frac{5}{8}x; \dots \right]$ or <b>SC:</b> $2 \left[ 1 + \dots - \frac{25}{128}x^2 + \dots \right]$ or <b>SC:</b> $\lambda \left[ 1 + \frac{5}{8}x - \frac{25}{128}x^2 + \dots \right]$ or <b>SC:</b> $\left[ \lambda + \frac{5\lambda}{8}x - \frac{25\lambda}{128}x^2 + \dots \right]$ (where $\lambda$ can be 1 or omitted), where each term in the $[\dots]$ is a simplified fraction or a decimal, <b>OR SC:</b> for $2 + \frac{10}{8}x - \frac{50}{128}x^2 + \dots$ (i.e. for not simplifying their correct coefficients.)
	<b>Note</b>	Candidates who write $2 \left[ 1 + \left(\frac{1}{2}\right)\left(-\frac{5x}{4}\right) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!} \left(-\frac{5x}{4}\right)^2 + \dots \right]$ , where $k = -\frac{5}{4}$ and not $\frac{5}{4}$ and achieve $2 - \frac{5}{4}x - \frac{25}{64}x^2 + \dots$ will get B1M1A1A0A1
	<b>Note</b>	Ignore extra terms beyond the term in $x^2$ .
	<b>Note</b>	You can ignore subsequent working following a correct answer.
	(b)	<b>B1</b> $\frac{3}{2}\sqrt{2}$ or $1.5\sqrt{2}$ or $k = \frac{3}{2}$ or $1.5$ o.e. (Ignore how $k = \frac{3}{2}$ is found.)
	(c)	<b>M1</b> Substitutes $x = \frac{1}{10}$ or $0.1$ into their binomial expansion found in part (a) which must contain both an $x$ term and an $x^2$ term (or even an $x^3$ term) <b>and</b> equates this to either $\frac{3}{\sqrt{2}}$ or their $k\sqrt{2}$ from (b), where $k$ is a numerical value.
<b>Note</b>	M1 can be implied by $\frac{3}{2}\sqrt{2}$ or $1.5\sqrt{2}$ or $\frac{3}{\sqrt{2}}$ = awrt 2.121	
<b>Note</b>	M1 <i>can be implied</i> by $\frac{1}{k} \left( \text{their } \frac{543}{256} \right)$ , with their $k$ found in part (b).	
<b>Note</b>	M1 <i>cannot be implied</i> by $(k) \left( \text{their } \frac{543}{256} \right)$ , with their $k$ found in part (b).	
<b>A1</b>	$\frac{181}{128}$ <b>or any equivalent fraction</b> , eg: $\frac{362}{256}$ or $\frac{543}{384}$ . Also allow $\frac{256}{181}$ <b>or any equivalent fraction</b> .	
<b>Note</b>	Also allow A1 for $p = 181, q = 128$ or $p = 181\lambda, q = 128\lambda$ or $p = 256, q = 181$ or $p = 256\lambda, q = 181\lambda$ , where $\lambda \in \mathbb{Z}^+$	
<b>Note</b>	You can recover work for part (c) in part (b). You cannot recover part (b) work in part (c).	
<b>Note</b>	Candidates are allowed to restart and gain all 2 marks in part (c) from an incorrect part (b).	
<b>Note</b>	Award M1 A1 for the correct answer from no working.	

1. (a)	<p><b>Alternative methods for part (a)</b></p> <p><b>Alternative method 1:</b> Candidates can apply an alternative form of the binomial expansion.</p> $\left\{ (4 + 5x)^{\frac{1}{2}} \right\} = (4)^{\frac{1}{2}} + \left(\frac{1}{2}\right)(4)^{-\frac{1}{2}}(5x) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(4)^{-\frac{3}{2}}(5x)^2$															
	<p><b>B1</b> <b>M1</b> <b>A1</b> <b>A1</b></p>	<p><math>(4)^{\frac{1}{2}}</math> or 2 Any two of three (un-simplified) terms correct. All three (un-simplified) terms correct. <math>2 + \frac{5}{4}x</math> (<b>simplified fractions</b>) or allow <math>2 + 1.25x</math> or <math>2 + 1\frac{1}{4}x</math> Accept only <math>-\frac{25}{64}x^2</math> or <math>-0.390625x^2</math></p>														
	<p><b>Note</b> The terms in C need to be evaluated. So <math>{}^{\frac{1}{2}}C_0(4)^{\frac{1}{2}} + {}^{\frac{1}{2}}C_1(4)^{-\frac{1}{2}}(5x) + {}^{\frac{1}{2}}C_2(4)^{-\frac{3}{2}}(5x)^2</math> without further working is BOM0A0.</p>															
	<p><b>Alternative Method 2: Maclaurin Expansion</b> <math>f(x) = (4 + 5x)^{\frac{1}{2}}</math></p> <table border="1" data-bbox="277 793 1521 1215"> <tr> <td data-bbox="277 793 1053 877"><math>f''(x) = -\frac{25}{4}(4 + 5x)^{-\frac{3}{2}}</math></td> <td data-bbox="1053 793 1373 877">Correct <math>f''(x)</math></td> <td data-bbox="1373 793 1521 877">B1</td> </tr> <tr> <td data-bbox="277 877 1053 1020" rowspan="2"><math>f'(x) = \frac{1}{2}(4 + 5x)^{-\frac{1}{2}}(5)</math></td> <td data-bbox="1053 877 1373 940"><math>\pm a(4 + 5x)^{-\frac{1}{2}}; a \neq \pm 1</math></td> <td data-bbox="1373 877 1521 940">M1</td> </tr> <tr> <td data-bbox="1053 940 1373 1020"><math>\frac{1}{2}(4 + 5x)^{-\frac{1}{2}}(5)</math></td> <td data-bbox="1373 940 1521 1020">A1 oe</td> </tr> <tr> <td data-bbox="277 1020 1053 1104"><math>\left\{ \therefore f(0) = 2, f'(0) = \frac{5}{4} \text{ and } f''(0) = -\frac{25}{32} \right\}</math></td> <td data-bbox="1053 1020 1373 1104"></td> <td data-bbox="1373 1020 1521 1104"></td> </tr> <tr> <td data-bbox="277 1104 1053 1215">So, <math>f(x) = 2 + \frac{5}{4}x; -\frac{25}{64}x^2 + \dots</math></td> <td data-bbox="1053 1104 1373 1215"></td> <td data-bbox="1373 1104 1521 1215">A1; A1</td> </tr> </table>		$f''(x) = -\frac{25}{4}(4 + 5x)^{-\frac{3}{2}}$	Correct $f''(x)$	B1	$f'(x) = \frac{1}{2}(4 + 5x)^{-\frac{1}{2}}(5)$	$\pm a(4 + 5x)^{-\frac{1}{2}}; a \neq \pm 1$	M1	$\frac{1}{2}(4 + 5x)^{-\frac{1}{2}}(5)$	A1 oe	$\left\{ \therefore f(0) = 2, f'(0) = \frac{5}{4} \text{ and } f''(0) = -\frac{25}{32} \right\}$			So, $f(x) = 2 + \frac{5}{4}x; -\frac{25}{64}x^2 + \dots$		A1; A1
$f''(x) = -\frac{25}{4}(4 + 5x)^{-\frac{3}{2}}$	Correct $f''(x)$	B1														
$f'(x) = \frac{1}{2}(4 + 5x)^{-\frac{1}{2}}(5)$	$\pm a(4 + 5x)^{-\frac{1}{2}}; a \neq \pm 1$	M1														
	$\frac{1}{2}(4 + 5x)^{-\frac{1}{2}}(5)$	A1 oe														
$\left\{ \therefore f(0) = 2, f'(0) = \frac{5}{4} \text{ and } f''(0) = -\frac{25}{32} \right\}$																
So, $f(x) = 2 + \frac{5}{4}x; -\frac{25}{64}x^2 + \dots$		A1; A1														

Question Number	Scheme		Marks
2.	$x^2 - 3xy - 4y^2 + 64 = 0$		
	(a)	$\left\{ \frac{dx}{dx} \right\} \times \left\{ \frac{dy}{dx} \right\} \times \left( 2x - \left( 3y + 3x \frac{dy}{dx} \right) - 8y \frac{dy}{dx} = 0 \right)$	M1A1 M1
		$2x - 3y + (-3x - 8y) \frac{dy}{dx} = 0$	dM1
		$\frac{dy}{dx} = \frac{2x - 3y}{3x + 8y}$ or $\frac{3y - 2x}{-3x - 8y}$	<b>o.e.</b> A1 cso
			[5]
	(b)	$\left\{ \frac{dy}{dx} = 0 \Rightarrow \right\} 2x - 3y = 0$	M1
		$y = \frac{2}{3}x$ $x = \frac{3}{2}y$	A1ft
		$x^2 - 3x\left(\frac{2}{3}x\right) - 4\left(\frac{2}{3}x\right)^2 + 64 = 0$ $\left(\frac{3}{2}y\right)^2 - 3\left(\frac{3}{2}y\right)y - 4y^2 + 64 = 0$	dM1
		$x^2 - 2x^2 - \frac{16}{9}x^2 + 64 = 0 \Rightarrow -\frac{25}{9}x^2 + 64 = 0$ $\frac{9}{4}y^2 - \frac{9}{2}y^2 - 4y^2 + 64 = 0 \Rightarrow -\frac{25}{4}y^2 + 64 = 0$	
		$\left\{ \Rightarrow x^2 = \frac{576}{25} \Rightarrow \right\} x = \frac{24}{5}$ or $-\frac{24}{5}$ $\left\{ \Rightarrow y^2 = \frac{256}{25} \Rightarrow \right\} y = \frac{16}{5}$ or $-\frac{16}{5}$	A1 cso
	When $x = \pm \frac{24}{5}$ , $y = \frac{2}{3}\left(\frac{24}{5}\right)$ and $-\frac{2}{3}\left(\frac{24}{5}\right)$ When $y = \pm \frac{16}{5}$ , $x = \frac{3}{2}\left(\frac{16}{5}\right)$ and $-\frac{3}{2}\left(\frac{16}{5}\right)$		
	$\left(\frac{24}{5}, \frac{16}{5}\right)$ and $\left(-\frac{24}{5}, -\frac{16}{5}\right)$ or $x = \frac{24}{5}$ , $y = \frac{16}{5}$ and $x = -\frac{24}{5}$ , $y = -\frac{16}{5}$	ddM1 cso A1	
		[6] 11	
(a)	<b>Alternative method for part (a)</b>		
		$\left\{ \frac{dx}{dx} \right\} \times \left\{ \frac{dy}{dy} \right\} \times \left( 2x \frac{dx}{dy} - \left( 3y \frac{dx}{dy} + 3x \right) - 8y = 0 \right)$	M1A1 M1
		$(2x - 3y) \frac{dx}{dy} - 3x - 8y = 0$	dM1
		$\frac{dy}{dx} = \frac{2x - 3y}{3x + 8y}$ or $\frac{3y - 2x}{-3x - 8y}$	<b>o.e.</b> A1 cso
		[5]	
<b>Question 2 Notes</b>			
2. (a) General	<b>Note</b>	Writing down $\frac{dy}{dx} = \frac{2x - 3y}{3x + 8y}$ or $\frac{3y - 2x}{-3x - 8y}$ from <b>no working</b> is full marks	
	<b>Note</b>	Writing down $\frac{dy}{dx} = \frac{2x - 3y}{-3x - 8y}$ or $\frac{3y - 2x}{3x + 8y}$ from <b>no working</b> is M1A0B1M1A0	
	<b>Note</b>	Few candidates will write $2x dx - 3y dx - 3x dy - 8y dy = 0$ leading to $\frac{dy}{dx} = \frac{2x - 3y}{3x + 8y}$ , o.e. This should get full marks.	

2. (a)	<b>M1</b>	Differentiates implicitly to include either $\pm 3x \frac{dy}{dx}$ or $-4y^2 \rightarrow \pm ky \frac{dy}{dx}$ . (Ignore $\left(\frac{dy}{dx} = \right)$ ).
	<b>A1</b>	<b>Both</b> $x^2 \rightarrow \underline{2x}$ <b>and</b> $\dots - 4y^2 + 64 = 0 \rightarrow -8y \frac{dy}{dx} = 0$
	<b>Note</b>	If an extra term appears then award A0.
	<b>M1</b>	$-3xy \rightarrow -3x \frac{dy}{dx} - 3y$ or $-3x \frac{dy}{dx} + 3y$ or $3x \frac{dy}{dx} - 3y$ or $3x \frac{dy}{dx} + 3y$
	<b>Note</b>	$2x - 3y - 3x \frac{dy}{dx} - 8y \frac{dy}{dx} \rightarrow 2x - 3y = 3x \frac{dy}{dx} + 8y \frac{dy}{dx}$ will get 1 <sup>st</sup> A1 (implied) as the " $= 0$ " can be implied by the rearrangement of their equation.
2. (b)	<b>dM1</b>	<b>dependent on the FIRST method mark being awarded.</b> An attempt to factorise out <b>all the terms</b> in $\frac{dy}{dx}$ as long as there are <b>at least two terms</b> in $\frac{dy}{dx}$ . i.e. $\dots + (-3x - 8y) \frac{dy}{dx} = \dots$ or $\dots = (3x + 8y) \frac{dy}{dx}$ . (Allow combining in 1 variable).
	<b>A1</b>	$\frac{2x - 3y}{3x + 8y}$ or $\frac{3y - 2x}{-3x - 8y}$ or equivalent.
	<b>Note</b>	<b>cso</b> If the candidate's solution is not completely correct, then do not give this mark.
	<b>Note</b>	You cannot recover work for part (a) in part (b).
	<b>M1</b>	Sets their numerator of their $\frac{dy}{dx}$ equal to zero (or the denominator of their $\frac{dx}{dy}$ equal to zero) o.e.
<b>Note</b>	1 <sup>st</sup> M1 can also be gained by setting $\frac{dy}{dx}$ equal to zero in their " $2x - 3y - 3x \frac{dy}{dx} - 8y \frac{dy}{dx} = 0$ "	
<b>Note</b>	<b>If their numerator involves one variable only then only the 1<sup>st</sup> M1 mark is possible in part (b).</b>	
<b>Note</b>	<b>If their numerator is a constant then no marks are available in part (b)</b>	
<b>Note</b>	<b>If their numerator is in the form <math>\pm ax^2 \pm by = 0</math> or <math>\pm ax \pm by^2 = 0</math> then the first 3 marks are possible in part (b).</b>	
<b>Note</b>	$\frac{dy}{dx} = \frac{2x - 3y}{3x + 8y} = 0$ is not sufficient for M1.	
<b>A1ft</b>	Either <ul style="list-style-type: none"> <li>Sets <math>2x - 3y</math> to zero and obtains either <math>y = \frac{2}{3}x</math> or <math>x = \frac{3}{2}y</math></li> <li>the follow through result of making either <math>y</math> or <math>x</math> the subject from setting their numerator of their <math>\frac{dy}{dx}</math> equal to zero</li> </ul>	
<b>dM1</b>	<b>dependent on the first method mark being awarded.</b> Substitutes <b>either</b> their $y = \frac{2}{3}x$ <b>or</b> their $x = \frac{3}{2}y$ into the original equation to give an equation in one variable only.	
<b>A1</b>	Obtains either $x = \frac{24}{5}$ or $-\frac{24}{5}$ or $y = \frac{16}{5}$ or $-\frac{16}{5}$ , (or equivalent) <b>by correct solution only.</b> i.e. You can allow for example $x = \frac{48}{10}$ or 4.8, etc.	
<b>Note</b>	$x = \sqrt{\frac{576}{25}}$ (not simplified) or $y = \sqrt{\frac{256}{25}}$ (not simplified) is not sufficient for A1.	

2. (b) ctd	ddM1	<p><b>dependent on both previous method marks being awarded in this part.</b></p> <p><b>Method 1</b></p> <p><b>Either:</b></p> <ul style="list-style-type: none"> <li>substitutes their <math>x</math> into their <math>y = \frac{2}{3}x</math> <b>or</b> substitutes their <math>y</math> into their <math>x = \frac{3}{2}y</math>, <b>or</b></li> <li>substitutes <i>the other of</i> their <math>y = \frac{2}{3}x</math> <b>or</b> their <math>x = \frac{3}{2}y</math> into the original equation,</li> </ul> <p><b>and achieves either:</b></p> <ul style="list-style-type: none"> <li>exactly two sets of two coordinates <b>or</b></li> <li>exactly two distinct values for <math>x</math> and exactly two distinct values for <math>y</math>.</li> </ul> <p><b>Method 2</b></p> <p><b>Either:</b></p> <ul style="list-style-type: none"> <li>substitutes their first <math>x</math>-value, <math>x_1</math> into <math>x^2 - 3xy - 4y^2 + 64 = 0</math> to obtain one <math>y</math>-value, <math>y_1</math> and substitutes their second <math>x</math>-value, <math>x_2</math> into <math>x^2 - 3xy - 4y^2 + 64 = 0</math> to obtain 1 <math>y</math>-value <math>y_2</math> <b>or</b></li> <li>substitutes their first <math>y</math>-value, <math>y_1</math> into <math>x^2 - 3xy - 4y^2 + 64 = 0</math> to obtain one <math>x</math>-value <math>x_1</math> and substitutes their second <math>y</math>-value, <math>y_2</math> into <math>x^2 - 3xy - 4y^2 + 64 = 0</math> to obtain one <math>x</math>-value <math>x_2</math>.</li> </ul>
<b>Note</b>		Three or more sets of coordinates given (without identification of two sets of coordinates) is ddM0.
<b>A1</b>	Both $\left(\frac{24}{5}, \frac{16}{5}\right)$ <b>and</b> $\left(-\frac{24}{5}, -\frac{16}{5}\right)$ , only <b>by cso</b> . Note that decimal equivalents are fine.	
<b>Note</b>	Also allow $x = \frac{24}{5}$ , $y = \frac{16}{5}$ <b>and</b> $x = -\frac{24}{5}$ , $y = -\frac{16}{5}$ all seen in their working to part (b).	
<b>Note</b>	Allow $x = \pm \frac{24}{5}$ , $y = \pm \frac{16}{5}$ for 3 <sup>rd</sup> A1.	
<b>Note</b>	$x = \pm \frac{24}{5}$ , $y = \pm \frac{16}{5}$ followed by eg. $\left(\frac{16}{5}, \frac{24}{5}\right)$ <b>and</b> $\left(-\frac{16}{5}, -\frac{24}{5}\right)$	
<b>Note</b>	(eg. coordinates stated the wrong way round) is 3 <sup>rd</sup> A0. It is possible for a candidate who does not achieve full marks in part (a), (but has a correct numerator for $\frac{dy}{dx}$ ) to gain all 6 marks in part (b).	
<b>Note</b>	Decimal equivalents to fractions are fine in part (b). i.e. $(4.8, 3.2)$ <b>and</b> $(-4.8, -3.2)$ .	
<b>Note</b>	$\left(\frac{24}{5}, \frac{16}{5}\right)$ <b>and</b> $\left(-\frac{24}{5}, -\frac{16}{5}\right)$ from no working is M0A0M0A0M0A0.	
<b>Note</b>	Candidates could potentially lose the final 2 marks for setting both their numerator and denominator to zero.	
<b>Note</b>	No credit in this part can be gained by only setting the denominator to zero.	

Question Number	Scheme		Marks	
3.	(a)	$y = 4x - xe^{\frac{1}{2}x}, x \geq 0$		
		$\left\{ y = 0 \Rightarrow 4x - xe^{\frac{1}{2}x} = 0 \Rightarrow x(4 - e^{\frac{1}{2}x}) = 0 \Rightarrow \right\}$		
		$e^{\frac{1}{2}x} = 4 \Rightarrow x_A = 4 \ln 2$	Attempts to solve $e^{\frac{1}{2}x} = 4$ giving $x = \dots$ in terms of $\pm \lambda \ln \mu$ where $\mu > 0$ $4 \ln 2$ <b>cao</b> (Ignore $x = 0$ )	
				[2]
	(b)	$\left\{ \int xe^{\frac{1}{2}x} dx \right\} = 2xe^{\frac{1}{2}x} - \int 2e^{\frac{1}{2}x} \{dx\}$	$\alpha xe^{\frac{1}{2}x} - \beta \int e^{\frac{1}{2}x} \{dx\}, \alpha > 0, \beta > 0$	M1
			$2xe^{\frac{1}{2}x} - \int 2e^{\frac{1}{2}x} \{dx\},$ with or without dx	A1 (M1 on ePEN)
		$= 2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x} \{+c\}$	$2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}$ o.e. with or without $+c$	A1
				[3]
	(c)	$\left\{ \int 4x dx \right\} = 2x^2$	$4x \rightarrow 2x^2$ or $\frac{4x^2}{2}$ o.e.	B1
		$\left\{ \int_0^{4 \ln 2} (4x - xe^{\frac{1}{2}x}) dx \right\} = \left[ 2x^2 - \left( 2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x} \right) \right]_0^{4 \ln 2 \text{ or } \ln 16 \text{ or their limits}}$		
$= \left( 2(4 \ln 2)^2 - 2(4 \ln 2)e^{\frac{1}{2}(4 \ln 2)} + 4e^{\frac{1}{2}(4 \ln 2)} \right) - \left( 2(0)^2 - 2(0)e^{\frac{1}{2}(0)} + 4e^{\frac{1}{2}(0)} \right)$		<b>See notes</b>	M1	
$= (32(\ln 2)^2 - 32(\ln 2) + 16) - (4)$ $= 32(\ln 2)^2 - 32(\ln 2) + 12$		$32(\ln 2)^2 - 32(\ln 2) + 12,$ <b>see notes</b>	A1	
			[3] 8	
<b>Question 3 Notes</b>				
3. (a)	<b>M1</b>	Attempts to solve $e^{\frac{1}{2}x} = 4$ giving $x = \dots$ in terms of $\pm \lambda \ln \mu$ where $\mu > 0$		
	<b>A1</b>	$4 \ln 2$ <b>cao</b> stated in part (a) only (Ignore $x = 0$ )		
(b)	<b>NOT E</b>	<b>Part (b) appears as M1M1A1 on ePEN, but is now marked as M1A1A1.</b>		
	<b>M1</b>	Integration by parts is applied in the form $\alpha xe^{\frac{1}{2}x} - \beta \int e^{\frac{1}{2}x} \{dx\},$ where $\alpha > 0, \beta > 0.$ (must be in this form) with or without dx		
	<b>A1</b>	$2xe^{\frac{1}{2}x} - \int 2e^{\frac{1}{2}x} \{dx\}$ or equivalent, with or without dx. <b>Can be un-simplified.</b>		
	<b>A1</b>	$2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}$ or equivalent with or without $+c.$ <b>Can be un-simplified.</b>		
	<b>Note</b>	You can also allow $2e^{\frac{1}{2}x}(x-2)$ or $e^{\frac{1}{2}x}(2x-4)$ for the final A1.		
	<b>isw</b>	You can ignore subsequent working following on from a correct solution.		
	<b>SC</b>	<b>SPECIAL CASE:</b> A candidate who uses $u = x, \frac{dv}{dx} = e^{\frac{1}{2}x},$ writes down the correct “by parts” formula, but makes only one error when applying it can be awarded Special Case M1. (Applying their $v$ counts for one consistent error.)		

3. (c)	<b>B1</b>	$4x \rightarrow 2x^2$ or $\frac{4x^2}{2}$ oe
	<b>M1</b>	<b>Complete</b> method of applying limits of their $x_A$ and 0 to all terms of an expression of the form $\pm Ax^2 \pm Bxe^{\frac{1}{2}x} \pm Ce^{\frac{1}{2}x}$ (where $A \neq 0$ , $B \neq 0$ and $C \neq 0$ ) and subtracting the correct way round.
	<b>Note</b>	Evidence of a proper consideration of the limit of 0 is needed for M1. So subtracting 0 is M0.
	<b>Note</b>	$\ln 16$ or $2 \ln 4$ or equivalent is fine as an upper limit.
	<b>A1</b>	A correct three term exact quadratic expression in $\ln 2$ . For example allow for A1 <ul style="list-style-type: none"> <li>• <math>32(\ln 2)^2 - 32(\ln 2) + 12</math></li> <li>• <math>8(2 \ln 2)^2 - 8(4 \ln 2) + 12</math></li> <li>• <math>2(4 \ln 2)^2 - 32(\ln 2) + 12</math></li> <li>• <math>2(4 \ln 2)^2 - 2(4 \ln 2)e^{\frac{1}{2}(4 \ln 2)} + 12</math></li> </ul>
	<b>Note</b>	Note that the constant term of 12 needs to be combined from $4e^{\frac{1}{2}(4 \ln 2)} - 4e^{\frac{1}{2}(0)}$ o.e.
	<b>Note</b>	Also allow $32 \ln 2(\ln 2 - 1) + 12$ or $32 \ln 2 \left( \ln 2 - 1 + \frac{12}{32 \ln 2} \right)$ for A1.
	<b>Note</b>	<b>Do not apply “ignore subsequent working” for incorrect simplification.</b> Eg: $32(\ln 2)^2 - 32(\ln 2) + 12 \rightarrow 64(\ln 2) - 32(\ln 2) + 12$ or $32(\ln 4) - 32(\ln 2) + 12$
	<b>Note</b>	<b>Bracketing error:</b> $32 \ln 2^2 - 32(\ln 2) + 12$ , unless recovered is final A0.
	<b>Note</b>	<b>Notation:</b> Allow $32(\ln^2 2) - 32(\ln 2) + 12$ for the final A1.
<b>Note</b>	5.19378... without seeing $32(\ln 2)^2 - 32(\ln 2) + 12$ is A0.	
<b>Note</b>	5.19378... following from a correct $2x^2 - \left( 2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x} \right)$ is M1A0.	
<b>Note</b>	5.19378... from no working is M0A0.	



Question Number	Scheme	Marks
4.	$l_1: \mathbf{r} = \begin{pmatrix} 5 \\ -3 \\ p \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}, \quad l_2: \mathbf{r} = \begin{pmatrix} 8 \\ 5 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}$ . Let $\theta$ = acute angle between $l_1$ and $l_2$ . <b>Note: You can mark parts (a) and (b) together.</b>	
(a)	$\{l_1 = l_2 \Rightarrow \mathbf{i}\}: 5 = 8 + 3\mu \Rightarrow \mu = -1$ Finds $\mu$ and substitutes their $\mu$ into $l_2$	M1
	So, $\{\overline{OA}\} = \begin{pmatrix} 8 \\ 5 \\ -2 \end{pmatrix} - 1 \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix}$ $5\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ or $\begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix}$ or $(5, 1, 3)$	A1
		[2]
(b)	$\{\mathbf{j}: -3 + \lambda = 5 + 4\mu \Rightarrow -3 + \lambda = 5 + 4(-1) \Rightarrow \lambda = 4$ Equates $\mathbf{j}$ components, substitutes their $\mu$ and solves to give $\lambda = \dots$	M1
	$\mathbf{k}: p - 3\lambda = -2 - 5\mu \Rightarrow$ $p - 3(4) = -2 - 5(-1) \Rightarrow \underline{p = 15}$ Equates $\mathbf{k}$ components, substitutes their $\lambda$ and their $\mu$ and solves to give $p = \dots$ <b>or</b> equates $\mathbf{k}$ components to give their " $p - 3\lambda =$ the $\mathbf{k}$ value of $A$ found in part (a)", substitutes their $\lambda$ and solves to give $p = \dots$	M1
	$\text{or } \mathbf{k}: p - 3\lambda = 3 \Rightarrow$ $p - 3(4) = 3 \Rightarrow \underline{p = 15}$ $p = 15$	A1
		[3]
(c)	$\mathbf{d}_1 = \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}, \quad \mathbf{d}_2 = \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}$ Realisation that the dot product is required between $\pm \mathbf{Ad}_1$ and $\pm \mathbf{Bd}_2$ .	M1
	$\cos \theta = \pm K \left( \frac{0(3) + (1)(4) + (-3)(-5)}{\sqrt{(0)^2 + (1)^2 + (-3)^2} \cdot \sqrt{(3)^2 + (4)^2 + (-5)^2}} \right)$ An attempt to apply the dot product formula between $\pm \mathbf{Ad}_1$ and $\pm \mathbf{Bd}_2$ .	dM1 (A1 on ePEN)
	$\cos \theta = \frac{19}{\sqrt{10} \cdot \sqrt{50}} \Rightarrow \theta = 31.8203116\dots = 31.82$ (2 dp) anything that rounds to 31.82	A1
		[3]
(d)	$\overline{OB} = \begin{pmatrix} 11 \\ 9 \\ -7 \end{pmatrix}; \quad \overline{AB} = \begin{pmatrix} 11 \\ 9 \\ -7 \end{pmatrix} - \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \\ -10 \end{pmatrix}$ or $\overline{AB} = 2 \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \\ -10 \end{pmatrix}$ See notes	M1
	$ \overline{AB}  = \sqrt{6^2 + 8^2 + (-10)^2} \{= 10\sqrt{2}\}$ Writes down a correct trigonometric equation involving the shortest distance, $d$ . Eg: $\frac{d}{\text{their } AB} = \sin \theta$ , oe.	dM1
	$\left\{ d = 10\sqrt{2} \sin 31.82\dots \Rightarrow \right\} d = 7.456540753\dots = 7.46$ (3sf) anything that rounds to 7.46	A1
		[3]
		11

4. (b)	<p><b>Alternative method for part (b)</b></p> $\begin{cases} 3 \times \mathbf{j}: -9 + 3\lambda = 15 + 12\mu \\ \mathbf{k}: p - 3\lambda = -2 + 5\mu \end{cases} \quad p - 9 = 13 + 7\mu$ $p - 9 = 13 + 7(-1) \Rightarrow \underline{p = 15}$	<p>Eliminates <math>\lambda</math> to write down an equation in <math>p</math> and <math>\mu</math></p> <p>Substitutes their <math>\mu</math> and solves to give</p> $p = \dots$ $p = 15$	<p>M1</p> <p>M1</p> <p>A1</p>
4. (d)	<p><b>Alternative Methods for part (d)</b> Let <math>X</math> be the foot of the perpendicular from <math>B</math> onto <math>l_1</math></p> $\mathbf{d}_1 = \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}, \quad \overrightarrow{OX} = \begin{pmatrix} 5 \\ -3 \\ 15 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix} = \begin{pmatrix} 5 \\ -3 + \lambda \\ 15 - 3\lambda \end{pmatrix}$ $\overrightarrow{BX} = \begin{pmatrix} 5 \\ -3 + \lambda \\ 15 - 3\lambda \end{pmatrix} - \begin{pmatrix} 11 \\ 9 \\ -7 \end{pmatrix} = \begin{pmatrix} -6 \\ -12 + \lambda \\ 22 - 3\lambda \end{pmatrix}$		
<b>Method 1</b>			
$\overrightarrow{BX} \cdot \mathbf{d}_1 = 0 \Rightarrow \begin{pmatrix} -6 \\ -12 + \lambda \\ 22 - 3\lambda \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix} = -12 + \lambda - 66 + 9\lambda = 0$ <p>leading to <math>10\lambda - 78 = 0 \Rightarrow \lambda = \frac{39}{5}</math></p>		<p>(Allow a sign slip in copying <math>\mathbf{d}_1</math>)</p> <p>Applies <math>\overrightarrow{BX} \cdot \mathbf{d}_1 = 0</math> and solves the resulting equation to find a value for <math>\lambda</math>.</p>	M1
$\overrightarrow{BX} = \begin{pmatrix} -6 \\ -12 + \frac{39}{5} \\ 22 - 3\left(\frac{39}{5}\right) \end{pmatrix} = \begin{pmatrix} -6 \\ -\frac{21}{5} \\ -\frac{7}{5} \end{pmatrix}$		<p>Substitutes their value of <math>\lambda</math> into their <math>\overrightarrow{BX}</math>.</p> <p><b>Note:</b> This mark is dependent upon the previous M1 mark.</p>	dM1
$d = BX = \sqrt{(-6)^2 + \left(-\frac{21}{5}\right)^2 + \left(-\frac{7}{5}\right)^2} = 7.456540753\dots$		awrt 7.46	A1
<b>Method 2</b>			
<p>Let <math>\beta =  \overrightarrow{BX} ^2 = 36 + 144 - 24\lambda + \lambda^2 + 484 - 132\lambda + 9\lambda^2</math></p> $= 10\lambda^2 - 156\lambda + 664$ <p>So <math>\frac{d\beta}{d\lambda} = 20\lambda - 156 = 0 \Rightarrow \lambda = \frac{39}{5}</math></p>		<p>Finds <math>\beta =  \overrightarrow{BX} ^2</math> in terms of <math>\lambda</math>,</p> <p>finds <math>\frac{d\beta}{d\lambda}</math> and sets this result equal to 0 and finds a value for <math>\lambda</math>.</p>	M1
$ \overrightarrow{BX} ^2 = 10\left(\frac{39}{5}\right)^2 - 156\left(\frac{39}{5}\right) + 664 = \frac{278}{5}$		<p>Substitutes their value of <math>\lambda</math> into their <math> \overrightarrow{BX} ^2</math>.</p> <p><b>Note:</b> This mark is dependent upon the previous M1 mark.</p>	dM1
$d = BX = \sqrt{\frac{278}{5}} = 7.456540753\dots$		awrt 7.46	A1

		<b>Question 4 Notes</b>	
4. (a)	<b>M1</b>	Finds $\mu$ and substitutes their $\mu$ into $l_2$	
	<b>A1</b>	Point of intersection of $5\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ . Allow $\begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix}$ or $(5, 1, 3)$ .	
	<b>Note</b>	You cannot recover the answer for part (a) in part (c) or part (d).	
(b)	<b>M1</b>	Equates <b>j</b> components, substitutes their $\mu$ and solves to give $\lambda = \dots$	
	<b>M1</b>	Equates <b>k</b> components, substitutes their $\lambda$ and their $\mu$ and solves to give $p = \dots$ or equates <b>k</b> components to give their " $p - 3\lambda =$ the <b>k</b> value of $A$ " found in part (b).	
	<b>A1</b>	$p = 15$	
(c)	<b>NOTE</b>	<b>Part (c) appears as M1A1A1 on ePEN, but now is marked as M1M1A1.</b>	
	<b>M1</b>	Realisation that the dot product is required between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$ .	
	<b>Note</b>	Allow one slip in candidates copying down their direction vectors, $\mathbf{d}_1$ and $\mathbf{d}_2$ .	
	<b>dM1</b>	<b>dependent on the FIRST method mark being awarded.</b> An attempt to apply the dot product formula between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$ .	
	<b>A1</b>	anything that rounds to 31.82. This can also be achieved by $180 - 148.1796\dots = \text{awrt } 31.82$	
	<b>Note</b>	$\theta = 0.5553\dots^\circ$ is A0.	
	<b>Note</b>	<b>M1A1 for</b> $\cos \theta = \left( \frac{0 - 16 - 60}{\sqrt{(0)^2 + (4)^2 + (-12)^2} \cdot \sqrt{(-3)^2 + (-4)^2 + (5)^2}} \right) = \frac{-76}{\sqrt{160} \cdot \sqrt{50}}$	
<b>Alternative Method: Vector Cross Product</b>			
<b>Only apply this scheme if it is clear that a candidate is applying a vector cross product method.</b>			
	$\mathbf{d}_1 \times \mathbf{d}_2 = \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix} \times \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} = \left\{ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & -3 \\ 3 & 4 & -5 \end{vmatrix} = 7\mathbf{i} - 9\mathbf{j} - 3\mathbf{k} \right\}$	Realisation that the vector cross product is required between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$ .	<b>M1</b>
	$\sin \theta = \frac{\sqrt{(7)^2 + (-9)^2 + (3)^2}}{\sqrt{(0)^2 + (1)^2 + (-3)^2} \cdot \sqrt{(3)^2 + (4)^2 + (-5)^2}}$	An attempt to apply the vector cross product formula	<b>dM1</b> (A1 on ePEN)
	$\sin \theta = \frac{\sqrt{139}}{\sqrt{10} \cdot \sqrt{50}} \Rightarrow \theta = 31.8203116\dots = 31.82$ (2 dp)	anything that rounds to 31.82	<b>A1</b>
(d)	<b>M1</b>	Full method for finding $B$ and for finding the magnitude of $\overline{AB}$ or the magnitude of $\overline{BA}$ .	
	<b>dM1</b>	<b>dependent on the first method mark being awarded.</b> Writes down correct trigonometric equation involving the shortest distance, $d$ . Eg: $\frac{d}{\text{their } AB} = \sin \theta$ or $\frac{d}{\text{their } AB} = \cos(90 - \theta)$ , o.e., where "their $AB$ " is a value. and $\theta =$ "their $\theta$ " or stated as $\theta$	
	<b>A1</b>	anything that rounds to 7.46	

Question Number	Scheme	Marks
5.	<b>Note: You can mark parts (a) and (b) together.</b>	
	(a) $x = 4t + 3, y = 4t + 8 + \frac{5}{2t}$	
	$\frac{dx}{dt} = 4, \frac{dy}{dt} = 4 - \frac{5}{2}t^{-2}$ Both $\frac{dx}{dt} = 4$ or $\frac{dt}{dx} = \frac{1}{4}$ and $\frac{dy}{dt} = 4 - \frac{5}{2}t^{-2}$	B1
	So, $\frac{dy}{dx} = \frac{4 - \frac{5}{2}t^{-2}}{4} \left\{ = 1 - \frac{5}{8}t^{-2} = 1 - \frac{5}{8t^2} \right\}$ Candidate's $\frac{dy}{dt}$ divided by a candidate's $\frac{dx}{dt}$	M1 o.e.
	{When $t = 2,$ } $\frac{dy}{dx} = \frac{27}{32}$ $\frac{27}{32}$ or 0.84375 <b>cao</b>	A1
		[3]
	<b>Way 2: Cartesian Method</b>	
	$\frac{dy}{dx} = 1 - \frac{10}{(x-3)^2}$ $\frac{dy}{dx} = 1 - \frac{10}{(x-3)^2}$ , simplified or un-simplified.	B1
		$\frac{dy}{dx} = \pm \lambda \pm \frac{\mu}{(x-3)^2}, \lambda \neq 0, \mu \neq 0$ M1
	{When $t = 2, x = 11$ } $\frac{dy}{dx} = \frac{27}{32}$ $\frac{27}{32}$ or 0.84375 <b>cao</b>	A1
	[3]	
<b>Way 3: Cartesian Method</b>		
$\frac{dy}{dx} = \frac{(2x+2)(x-3) - (x^2+2x-5)}{(x-3)^2}$ Correct expression for $\frac{dy}{dx}$ , simplified or un-simplified.	B1	
$\left\{ = \frac{x^2 - 6x - 1}{(x-3)^2} \right\}$ $\frac{dy}{dx} = \frac{f'(x)(x-3) - f(x)}{(x-3)^2}$ , where $f(x) = \text{their } "x^2 + ax + b"$ , $g(x) = x - 3$	M1	
{When $t = 2, x = 11$ } $\frac{dy}{dx} = \frac{27}{32}$ $\frac{27}{32}$ or 0.84375 <b>cao</b>	A1	
	[3]	
(b) $\left\{ t = \frac{x-3}{4} \Rightarrow \right\} y = 4\left(\frac{x-3}{4}\right) + 8 + \frac{5}{2\left(\frac{x-3}{4}\right)}$ Eliminates $t$ to achieve an equation in only $x$ and $y$	M1	
$y = x - 3 + 8 + \frac{10}{x-3}$		
$y = \frac{(x-3)(x-3) + 8(x-3) + 10}{x-3}$ or $y(x-3) = (x-3)(x-3) + 8(x-3) + 10$ See notes	dM1	
or $y = \frac{(x+5)(x-3) + 10}{x-3}$ or $y = \frac{(x+5)(x-3)}{x-3} + \frac{10}{x-3}$		
$\Rightarrow y = \frac{x^2 + 2x - 5}{x-3}, \{a = 2 \text{ and } b = -5\}$ Correct algebra leading to $y = \frac{x^2 + 2x - 5}{x-3}$ or $a = 2$ and $b = -5$	A1 cso	
	[3]	
	6	

Question Number	Scheme	Marks
5. (b)	<b>Alternative Method 1 of Equating Coefficients</b>	
	$y = \frac{x^2 + ax + b}{x - 3} \Rightarrow y(x - 3) = x^2 + ax + b$ $y(x - 3) = (4t + 3)^2 + 2(4t + 3) - 5 = 16t^2 + 32t + 10$ $x^2 + ax + b = (4t + 3)^2 + a(4t + 3) + b$	
	$(4t + 3)^2 + a(4t + 3) + b = 16t^2 + 32t + 10$	Correct method of obtaining an equation in only $t$ , $a$ and $b$
	$t: \quad 24 + 4a = 32 \Rightarrow a = 2$	Equates their coefficients in $t$ and finds both $a = \dots$ and $b = \dots$
	$\text{constant: } 9 + 3a + b = 10 \Rightarrow b = -5$	$a = 2 \text{ and } b = -5$
5. (b)	<b>Alternative Method 2 of Equating Coefficients</b>	
	$\left\{ t = \frac{x - 3}{4} \Rightarrow \right\} y = 4 \left( \frac{x - 3}{4} \right) + 8 + \frac{5}{2 \left( \frac{x - 3}{4} \right)}$	Eliminates $t$ to achieve an equation in only $x$ and $y$
	$y = x - 3 + 8 + \frac{10}{x - 3} \Rightarrow y = x + 5 + \frac{10}{x - 3}$	
	$\underline{y(x - 3)} = (x + 5)(x - 3) + 10 \Rightarrow \underline{x^2 + ax + b} = \underline{(x + 5)(x - 3) + 10}$	dM1
	$\Rightarrow y = \frac{x^2 + 2x - 5}{x - 3}$	<p>Correct algebra leading to</p> <p>or equating coefficients to give <math>a = 2</math> and <math>b = -5</math></p> $y = \frac{x^2 + 2x - 5}{x - 3} \text{ or } a = 2 \text{ and } b = -5$ <p>A1 cso</p>

[3]

[3]

		<b>Question 5 Notes</b>
5. (a)	<b>B1</b>	$\frac{dx}{dt} = 4$ and $\frac{dy}{dt} = 4 - \frac{5}{2}t^{-2}$ or $\frac{dy}{dt} = \frac{8t^2 - 5}{2t^2}$ or $\frac{dy}{dt} = 4 - 5(2t)^{-2}(2)$ , etc.
	<b>Note</b>	$\frac{dy}{dt}$ can be simplified or un-simplified.
	<b>Note</b>	<b>You can imply the B1 mark by later working.</b>
	<b>M1</b>	Candidate's $\frac{dy}{dt}$ divided by a candidate's $\frac{dx}{dt}$ or $\frac{dy}{dt}$ multiplied by a candidate's $\frac{dt}{dx}$
	<b>Note</b>	M1 can be also be obtained by substituting $t = 2$ into both their $\frac{dy}{dt}$ and their $\frac{dx}{dt}$ and then dividing their values the correct way round.
	<b>A1</b>	$\frac{27}{32}$ or 0.84375 <b>cao</b>
(b)	<b>M1</b>	Eliminates $t$ to achieve an equation in only $x$ and $y$ .
	<b>dM1</b>	<b>dependent on the first method mark being awarded.</b> <b>Either: (ignoring sign slips or constant slips, noting that <math>k</math> can be 1)</b>
		<ul style="list-style-type: none"> <li>Combining all three parts of their <math>\underline{x-3} + \bar{8} + \left(\frac{10}{\underline{x-3}}\right)</math> to form a single fraction with a common denominator of <math>\pm k(x-3)</math>. Accept three separate fractions with the same denominator.</li> <li>Combining both parts of their <math>\underline{x+5} + \left(\frac{10}{\underline{x-3}}\right)</math>, (where <math>\underline{x+5}</math> is their <math>4\left(\frac{x-3}{4}\right) + 8</math>), to form a single fraction with a common denominator of <math>\pm k(x-3)</math>. Accept two separate fractions with the same denominator.</li> <li>Multiplies both sides of their <math>y = \underline{x-3} + \bar{8} + \left(\frac{10}{\underline{x-3}}\right)</math> or their <math>y = \underline{x+5} + \left(\frac{10}{\underline{x-3}}\right)</math> by <math>\pm k(x-3)</math>. Note that all terms in their equation must be multiplied by <math>\pm k(x-3)</math>.</li> </ul>
	<b>Note</b>	Condone "invisible" brackets for dM1.
	<b>A1</b>	Correct algebra with no incorrect working leading to $y = \frac{x^2 + 2x - 5}{x - 3}$ or $a = 2$ and $b = -5$
	<b>Note</b>	<b>Some examples for the award of dM1 in (b):</b> <b>dM0</b> for $y = x - 3 + 8 + \frac{10}{x - 3} \rightarrow y = \frac{(x - 3)(x - 3) + 8 + 10}{x - 3}$ . Should be $\dots + 8(x - 3) + \dots$ <b>dM0</b> for $y = x - 3 + \frac{10}{x - 3} \rightarrow y = \frac{(x - 3)(x - 3) + 10}{x - 3}$ . The "8" part has been omitted. <b>dM0</b> for $y = x + 5 + \frac{10}{x - 3} \rightarrow y = \frac{x(x - 3) + 5 + 10}{x - 3}$ . Should be $\dots + 5(x - 3) + \dots$ <b>dM0</b> for $y = x + 5 + \frac{10}{x - 3} \rightarrow y(x - 3) = x(x - 3) + 5(x - 3) + 10(x - 3)$ . Should be just 10.
<b>Note</b>	$y = x + 5 + \frac{10}{x - 3} \rightarrow y = \frac{x^2 + 2x - 5}{x - 3}$ with no intermediate working is dM1A1.	

Question Number	Scheme	Marks	
6. (a)	$A = \int_0^3 \sqrt{(3-x)(x+1)} dx, x = 1 + 2\sin\theta$		
	$\frac{dx}{d\theta} = 2\cos\theta$	$\frac{dx}{d\theta} = 2\cos\theta$ or $2\cos\theta$ used correctly in their working. Can be implied.	B1
	$\left\{ \int \sqrt{(3-x)(x+1)} dx \text{ or } \int \sqrt{(3+2x-x^2)} dx \right\}$		
	$= \int \sqrt{(3-(1+2\sin\theta))(1+2\sin\theta+1)} 2\cos\theta \{d\theta\}$	Substitutes for both $x$ and $dx$ , where $dx \neq \lambda d\theta$ . Ignore $d\theta$	M1
	$= \int \sqrt{(2-2\sin\theta)(2+2\sin\theta)} 2\cos\theta \{d\theta\}$		
	$= \int \sqrt{(4-4\sin^2\theta)} 2\cos\theta \{d\theta\}$		
	$= \int \sqrt{(4-4(1-\cos^2\theta))} 2\cos\theta \{d\theta\} \text{ or } \int \sqrt{4\cos^2\theta} 2\cos\theta \{d\theta\}$	Applies $\cos^2\theta = 1 - \sin^2\theta$ <b>see notes</b>	M1
	$= 4 \int \cos^2\theta d\theta, \{k=4\}$	$4 \int \cos^2\theta d\theta$ or $\int 4\cos^2\theta d\theta$ <b>Note:</b> $d\theta$ is required here.	A1
	$0 = 1 + 2\sin\theta$ or $-1 = 2\sin\theta$ or $\sin\theta = -\frac{1}{2} \Rightarrow \theta = -\frac{\pi}{6}$	See notes	B1
	<b>and</b> $3 = 1 + 2\sin\theta$ or $2 = 2\sin\theta$ or $\sin\theta = 1 \Rightarrow \theta = \frac{\pi}{2}$		
		[5]	
(b)	$\left\{ k \int \cos^2\theta \{d\theta\} \right\} = \left\{ k \int \left( \frac{1+\cos 2\theta}{2} \right) \{d\theta\} \right\}$	Applies $\cos 2\theta = 2\cos^2\theta - 1$ to their integral	M1
	$= \left\{ k \left( \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta \right) \right\}$	Integrates to give $\pm\alpha\theta \pm \beta\sin 2\theta, \alpha \neq 0, \beta \neq 0$ or $k(\pm\alpha\theta \pm \beta\sin 2\theta)$	M1 (A1 on ePEN)
	$\left\{ \text{So } 4 \int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta d\theta = \left[ 2\theta + \sin 2\theta \right]_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \right\}$		
	$= \left( 2\left(\frac{\pi}{2}\right) + \sin\left(\frac{2\pi}{2}\right) \right) - \left( 2\left(-\frac{\pi}{6}\right) + \sin\left(-\frac{2\pi}{6}\right) \right)$		
	$\left\{ = \left( \pi \right) - \left( -\frac{\pi}{3} - \frac{\sqrt{3}}{2} \right) \right\} = \frac{4\pi}{3} + \frac{\sqrt{3}}{2}$	$\frac{4\pi}{3} + \frac{\sqrt{3}}{2}$ or $\frac{1}{6}(8\pi + 3\sqrt{3})$	A1 cao cso
		[3] 8	

<b>Question 6 Notes</b>	
<b>6. (a)</b>	<p><b>B1</b> <math>\frac{dx}{d\theta} = 2\cos\theta</math>. Also allow <math>dx = 2\cos\theta d\theta</math>. This mark can be implied by later working.</p> <p><b>Note</b> You can give B1 for <math>2\cos\theta</math> used correctly in their working.</p> <p><b>M1</b> Substitutes <math>x = 1 + 2\sin\theta</math> and their <math>dx</math> (from their rearranged <math>\frac{dx}{d\theta}</math>) into <math>\sqrt{(3-x)(x+1)} dx</math>.</p> <p><b>Note</b> Condone bracketing errors here.</p> <p><b>Note</b> <math>dx \neq \lambda d\theta</math>. For example <math>dx \neq d\theta</math>.</p> <p><b>Note</b> Condone substituting <math>dx = \cos\theta</math> for the 1<sup>st</sup> M1 after a correct <math>\frac{dx}{d\theta} = 2\cos\theta</math> or <math>dx = 2\cos\theta d\theta</math></p>
	<p><b>M1</b> Applies either</p> <ul style="list-style-type: none"> <li>• <math>1 - \sin^2\theta = \cos^2\theta</math></li> <li>• <math>\lambda - \lambda\sin^2\theta</math> or <math>\lambda(1 - \sin^2\theta) = \lambda\cos^2\theta</math></li> <li>• <math>4 - 4\sin^2\theta = 4 + 2\cos 2\theta - 2 = 2 + 2\cos 2\theta = 4\cos^2\theta</math></li> </ul> <p>to their expression where <math>\lambda</math> is a numerical value.</p>
	<p><b>A1</b> Correctly proves that <math>\int \sqrt{(3-x)(x+1)} dx</math> is equal to <math>4\int \cos^2\theta d\theta</math> or <math>\int 4\cos^2\theta d\theta</math></p> <p><b>Note</b> All three previous marks must have been awarded before A1 can be awarded.</p> <p><b>Note</b> Their final answer must include <math>d\theta</math>.</p> <p><b>Note</b> You can ignore limits for the final A1 mark.</p>
	<p><b>B1</b> Evidence of a correct equation in <math>\sin\theta</math> or <math>\sin^{-1}\theta</math> for both <math>x</math>-values leading to both <math>\theta</math> values. Eg:</p> <ul style="list-style-type: none"> <li>• <math>0 = 1 + 2\sin\theta</math> or <math>-1 = 2\sin\theta</math> or <math>\sin\theta = -\frac{1}{2}</math> which then leads to <math>\theta = -\frac{\pi}{6}</math>, <b>and</b></li> <li>• <math>3 = 1 + 2\sin\theta</math> or <math>2 = 2\sin\theta</math> or <math>\sin\theta = 1</math> which then leads to <math>\theta = \frac{\pi}{2}</math></li> </ul>
	<p><b>Note</b> Allow B1 for <math>x = 1 + 2\sin\left(-\frac{\pi}{6}\right) = 0</math> <b>and</b> <math>x = 1 + 2\sin\left(\frac{\pi}{2}\right) = 3</math></p>
	<p><b>Note</b> Allow B1 for <math>\sin\theta = \left(\frac{x-1}{2}\right)</math> or <math>\theta = \sin^{-1}\left(\frac{x-1}{2}\right)</math> followed by <math>x=0, \theta = -\frac{\pi}{6}; x=3, \theta = \frac{\pi}{2}</math></p>
	<p><b>(b) NOTE</b> <b>Part (b) appears as M1A1A1 on ePEN, but is now marked as M1M1A1.</b></p>
	<p><b>M1</b> Writes down a correct equation involving <math>\cos 2\theta</math> and <math>\cos^2\theta</math></p> <p><b>Eg:</b> <math>\cos 2\theta = 2\cos^2\theta - 1</math> or <math>\cos^2\theta = \frac{1 + \cos 2\theta}{2}</math> or <math>\lambda\cos^2\theta = \lambda\left(\frac{1 + \cos 2\theta}{2}\right)</math></p> <p>and <b>applies</b> it to their integral. <b>Note:</b> Allow M1 for a correctly stated formula (via an incorrect rearrangement) being applied to their integral.</p>
	<p><b>M1</b> Integrates to give an expression of the form <math>\pm\alpha\theta \pm \beta\sin 2\theta</math> or <math>k(\pm\alpha\theta \pm \beta\sin 2\theta)</math>, <math>\alpha \neq 0, \beta \neq 0</math> (can be simplified or un-simplified).</p>
	<p><b>A1</b> A <b>correct solution in part (b)</b> leading to a “two term” exact answer.</p> <p>Eg: <math>\frac{4\pi}{3} + \frac{\sqrt{3}}{2}</math> <b>or</b> <math>\frac{8\pi}{6} + \frac{\sqrt{3}}{2}</math> <b>or</b> <math>\frac{1}{6}(8\pi + 3\sqrt{3})</math></p>
<p><b>Note</b> 5.054815... from no working is M0M0A0.</p> <p><b>Note</b> Candidates can work in terms of <math>k</math> (note that <math>k</math> is not given in (a)) for the M1M1 marks in part (b).</p>	
<p><b>Note</b> If they incorrectly obtain <math>4\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta d\theta</math> in part (a) (or guess <math>k = 4</math>) then the final A1 is available for a correct solution in part (b) only.</p>	



Question Number	Scheme	Marks
7. (a)	$\frac{2}{P(P-2)} = \frac{A}{P} + \frac{B}{(P-2)}$	
	$2 = A(P-2) + BP$	Can be implied. M1
	$A = -1, B = 1$	Either one. A1
	giving $\frac{1}{(P-2)} - \frac{1}{P}$	See notes. <b>cao, aef</b> A1
		[3]
(b)	$\frac{dP}{dt} = \frac{1}{2}P(P-2)\cos 2t$	
	$\int \frac{2}{P(P-2)} dP = \int \cos 2t dt$	can be implied by later working B1 oe
	$\ln(P-2) - \ln P = \frac{1}{2}\sin 2t (+c)$	$\pm \lambda \ln(P-2) \pm \mu \ln P,$ $\lambda \neq 0, \mu \neq 0$ M1
	$\ln(P-2) - \ln P = \frac{1}{2}\sin 2t$	A1
	$\{t = 0, P = 3 \Rightarrow\} \ln 1 - \ln 3 = 0 + c \quad \{\Rightarrow c = -\ln 3 \text{ or } \ln(\frac{1}{3})\}$	See notes M1
	$\ln(P-2) - \ln P = \frac{1}{2}\sin 2t - \ln 3$	
	$\ln\left(\frac{3(P-2)}{P}\right) = \frac{1}{2}\sin 2t$	
	$\frac{3(P-2)}{P} = e^{\frac{1}{2}\sin 2t}$	Starting from an equation of the form $\pm \lambda \ln(P-\beta) \pm \mu \ln P = \pm K \sin \delta t + c,$ $\lambda, \mu, \beta, K, \delta \neq 0,$ applies a fully correct method to eliminate their logarithms. <b>Must have a constant of integration that need not be evaluated (see note)</b> M1
	$3(P-2) = Pe^{\frac{1}{2}\sin 2t} \Rightarrow 3P - 6 = Pe^{\frac{1}{2}\sin 2t}$	A complete method of rearranging to make $P$ the subject.
	gives $3P - Pe^{\frac{1}{2}\sin 2t} = 6 \Rightarrow P(3 - e^{\frac{1}{2}\sin 2t}) = 6$	<b>Must have a constant of integration that need not be evaluated (see note)</b> dM1
$P = \frac{6}{(3 - e^{\frac{1}{2}\sin 2t})} *$	Correct proof. A1 * cso	
		[7]
(c)	$\{\text{population} = 4000 \Rightarrow\} P = 4$	States $P = 4$ or applies $P = 4$ M1
	$\frac{1}{2}\sin 2t = \ln\left(\frac{3(4-2)}{4}\right) \left\{ = \ln\left(\frac{3}{2}\right) \right\}$	Obtains $\pm \lambda \sin 2t = \ln k$ or $\pm \lambda \sin t = \ln k,$ $\lambda \neq 0, k > 0$ where $\lambda$ and $k$ are numerical values and $\lambda$ can be 1 M1
	$t = 0.4728700467\dots$	anything that rounds to 0.473 Do not apply isw here A1
		[3] 13

Question Number	Scheme		Marks
7. (b)	<b>Method 2 for Q7(b)</b>		
	$\ln(P-2) - \ln P = \frac{1}{2} \sin 2t + c$	As before for...	<b>B1M1A1</b>
	$\ln\left(\frac{P-2}{P}\right) = \frac{1}{2} \sin 2t + c$		
	$\frac{P-2}{P} = e^{\frac{1}{2} \sin 2t + c}$ or $\frac{P-2}{P} = Ae^{\frac{1}{2} \sin 2t}$	Starting from an equation of the form $\pm \lambda \ln(P-\beta) \pm \mu \ln P = \pm K \sin \delta t + c$ , $\lambda, \mu, \beta, K, \delta \neq 0$ , applies a fully correct method to eliminate their logarithms. <b>Must have a constant of integration that need not be evaluated (see note)</b>	<b>3<sup>rd</sup> M1</b>
	$(P-2) = APe^{\frac{1}{2} \sin 2t} \Rightarrow P - APe^{\frac{1}{2} \sin 2t} = 2$	A complete method of rearranging to make $P$ the subject. Condone sign slips or constant errors. <b>Must have a constant of integration that need not be evaluated (see note)</b>	<b>4<sup>th</sup> dM1</b>
	$\Rightarrow P(1 - Ae^{\frac{1}{2} \sin 2t}) = 2 \Rightarrow P = \frac{2}{(1 - Ae^{\frac{1}{2} \sin 2t})}$	See notes <b>(Allocate this mark as the 2<sup>nd</sup> M1 mark on ePEN).</b>	<b>2<sup>nd</sup> M1</b>
	$\{t = 0, P = 3 \Rightarrow\} \quad 3 = \frac{2}{(1 - Ae^{\frac{1}{2} \sin 2(0)})}$		
$\left\{ \Rightarrow 3 = \frac{2}{(1-A)} \Rightarrow A = \frac{1}{3} \right\}$			
$\Rightarrow P = \frac{2}{\left(1 - \frac{1}{3}e^{\frac{1}{2} \sin 2t}\right)} \Rightarrow P = \frac{6}{(3 - e^{\frac{1}{2} \sin 2t})}^*$	Correct proof.	<b>A1 * cso</b>	
<b>Question 7 Notes</b>			
7. (a)	<b>M1</b>	Forming a correct identity. For example, $2 \equiv A(P-2) + BP$ from $\frac{2}{P(P-2)} = \frac{A}{P} + \frac{B}{(P-2)}$	
	<b>Note</b>	$A$ and $B$ are not referred to in question.	
	<b>A1</b>	Either one of $A = -1$ or $B = 1$ .	
	<b>A1</b>	$\frac{1}{(P-2)} - \frac{1}{P}$ or any equivalent form. This answer <i>cannot</i> be recovered from part (b).	
	<b>Note</b>	M1A1A1 can also be given for a candidate who finds both $A = -1$ and $B = 1$ and $\frac{A}{P} + \frac{B}{(P-2)}$ is seen in their working.	
<b>Note</b>	Candidates can use 'cover-up' rule to write down $\frac{1}{(P-2)} - \frac{1}{P}$ , so as to gain all three marks.		
<b>Note</b>	Equating coefficients from $2 \equiv A(P-2) + BP$ gives $A+B=2, -2A=2 \Rightarrow A=-1, B=1$		

7. (b)	<b>B1</b>	Separates variables as shown on the Mark Scheme. $dP$ and $dt$ should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.
	<b>Note</b>	<b>Eg:</b> $\int \frac{2}{P^2 - 2P} dP = \int \cos 2t dt$ or $\int \frac{1}{P(P-2)} dP = \frac{1}{2} \int \cos 2t dt$ <b>o.e.</b> are also fine for B1.
	<b>1<sup>st</sup> M1</b>	$\pm \lambda \ln(P-2) \pm \mu \ln P$ , $\lambda \neq 0, \mu \neq 0$ . Also allow $\pm \lambda \ln(M(P-2)) \pm \mu \ln NP$ ; $M, N$ can be 1.
	<b>Note</b>	Condone $2 \ln(P-2) + 2 \ln P$ or $2 \ln(P(P-2))$ or $2 \ln(P^2 - 2P)$ or $\ln(P^2 - 2P)$
	<b>1<sup>st</sup> A1</b>	Correct result of $\ln(P-2) - \ln P = \frac{1}{2} \sin 2t$ or $2 \ln(P-2) - 2 \ln P = \sin 2t$ o.e. with or without $+c$
	<b>2<sup>nd</sup> M1</b>	Some evidence of using both $t=0$ and $P=3$ in an integrated equation containing a constant of integration. Eg: $c$ or $A$ , etc.
	<b>3<sup>rd</sup> M1</b>	Starting from an equation of the form $\pm \lambda \ln(P-\beta) \pm \mu \ln P = \pm K \sin \delta t + c$ , $\lambda, \mu, \beta, K, \delta \neq 0$ , applies a fully correct method to eliminate their logarithms.
	<b>4<sup>th</sup> M1</b>	<b>dependent on the third method mark being awarded.</b> A complete method of rearranging to make $P$ the subject. Condone sign slips or constant errors.
	<b>Note</b>	For the 3 <sup>rd</sup> M1 and 4 <sup>th</sup> M1 marks, a candidate needs to have included a constant of integration, in their working. eg. $c, A, \ln A$ or an evaluated constant of integration.
	<b>2<sup>nd</sup> A1</b>	Correct proof of $P = \frac{6}{(3 - e^{\frac{1}{2} \sin 2t})}$ . <b>Note:</b> This answer is given in the question.
<b>Note</b>	$\ln\left(\frac{P-2}{P}\right) = \frac{1}{2} \sin 2t + c$ followed by $\frac{P-2}{P} = e^{\frac{1}{2} \sin 2t} + e^c$ is 3 <sup>rd</sup> M0, 4 <sup>th</sup> M0, 2 <sup>nd</sup> A0.	
<b>Note</b>	$\ln\left(\frac{P-2}{P}\right) = \frac{1}{2} \sin 2t + c \rightarrow \frac{P-2}{P} = e^{\frac{1}{2} \sin 2t + c} \rightarrow \frac{P-2}{P} = e^{\frac{1}{2} \sin 2t} + e^c$ is final M1M0A0	
<b>4<sup>th</sup> M1 for making P the subject</b>		
<b>Note there are three type of manipulations here which are considered acceptable for making P the subject.</b>		
(1) M1 for $\frac{3(P-2)}{P} = e^{\frac{1}{2} \sin 2t} \Rightarrow 3(P-2) = P e^{\frac{1}{2} \sin 2t} \Rightarrow 3P - 6 = P e^{\frac{1}{2} \sin 2t} \Rightarrow P(3 - e^{\frac{1}{2} \sin 2t}) = 6$ $\Rightarrow P = \frac{6}{(3 - e^{\frac{1}{2} \sin 2t})}$		
(2) M1 for $\frac{3(P-2)}{P} = e^{\frac{1}{2} \sin 2t} \Rightarrow 3 - \frac{6}{P} = e^{\frac{1}{2} \sin 2t} \Rightarrow 3 - e^{\frac{1}{2} \sin 2t} = \frac{6}{P} \Rightarrow P = \frac{6}{(3 - e^{\frac{1}{2} \sin 2t})}$		
(3) M1 for $\left\{ \ln(P-2) + \ln P = \frac{1}{2} \sin 2t + \ln 3 \Rightarrow \right\} P(P-2) = 3e^{\frac{1}{2} \sin 2t} \Rightarrow P^2 - 2P = 3e^{\frac{1}{2} \sin 2t}$ $\Rightarrow (P-1)^2 - 1 = 3e^{\frac{1}{2} \sin 2t}$ leading to $P = ..$		
(c)	<b>M1</b>	States $P = 4$ or applies $P = 4$
	<b>M1</b>	Obtains $\pm \lambda \sin 2t = \ln k$ or $\pm \lambda \sin t = \ln k$ , where $\lambda$ and $k$ are numerical values and $\lambda$ can be 1
	<b>A1</b>	anything that rounds to 0.473. ( <b>Do not apply isw here</b> )
	<b>Note</b>	<b>Do not apply ignore subsequent working for A1.</b> (Eg: 0.473 followed by 473 years is A0.)
	<b>Note</b>	Use of $P = 4000$ : Without the mention of $P = 4$ , $\frac{1}{2} \sin 2t = \ln 2.9985$ or $\sin 2t = 2 \ln 2.9985$ or $\sin 2t = 2.1912...$ will usually imply M0M1A0
<b>Note</b>	Use of Degrees: $t = \text{awrt } 27.1$ will usually imply M1M1A0	

Question Number	Scheme	Marks	
8. (a)	$\{y = 3^x \Rightarrow \frac{dy}{dx} = 3^x \ln 3\}$ $\frac{dy}{dx} = 3^x \ln 3$ or $\ln 3(e^{x \ln 3})$ or $y \ln 3$	B1	
	<b>Either T:</b> $y - 9 = 3^2 \ln 3(x - 2)$ <span style="float: right;">See notes</span>	M1	
	<b>or T:</b> $y = (3^2 \ln 3)x + 9 - 18 \ln 3$ , where $9 = (3^2 \ln 3)(2) + c$		
	{Cuts $x$ -axis $\Rightarrow y = 0 \Rightarrow$ }		
	$-9 = 9 \ln 3(x - 2)$ or $0 = (3^2 \ln 3)x + 9 - 18 \ln 3$ , <span style="float: right;">Sets <math>y = 0</math> in their tangent equation and progresses to <math>x = \dots</math></span>	M1	
	So, $x = 2 - \frac{1}{\ln 3}$ <span style="float: right;"><math>2 - \frac{1}{\ln 3}</math> or <math>\frac{2 \ln 3 - 1}{\ln 3}</math> o.e.</span>	A1 cso	
		<b>[4]</b>	
	(b) $V = \pi \int (3^x)^2 \{dx\}$ or $\pi \int 3^{2x} \{dx\}$ or $\pi \int 9^x \{dx\}$ <span style="float: right;"><math>V = \pi \int (3^x)^2</math> with or without <math>dx</math>, which can be implied</span>	B1 o.e.	
	$= \{\pi\} \left\{ \frac{3^{2x}}{2 \ln 3} \right\}$ or $= \{\pi\} \left\{ \frac{9^x}{\ln 9} \right\}$ <span style="float: right;">Eg: either <math>3^{2x} \rightarrow \frac{3^{2x}}{\pm \alpha (\ln 3)}</math> or <math>\pm \alpha (\ln 3) 3^{2x}</math> or <math>9^x \rightarrow \frac{9^x}{\pm \alpha (\ln 9)}</math> or <math>\pm \alpha (\ln 9) 9^x</math>, <math>\alpha \in \mathbb{R}</math></span>	M1	
	$3^{2x} \rightarrow \frac{3^{2x}}{2 \ln 3}$ or $9^x \rightarrow \frac{9^x}{\ln 9}$ or $e^{2x \ln 3} \rightarrow \frac{1}{2 \ln 3} (e^{2x \ln 3})$ <span style="float: right;"><b>Dependent on the previous method mark.</b> Substitutes <math>x = 2</math> and <math>x = 0</math> and subtracts the correct way round.</span>	A1 o.e.	
$\left\{ V = \pi \int_0^2 3^{2x} dx = \{\pi\} \left[ \frac{3^{2x}}{2 \ln 3} \right]_0^2 \right\} = \{\pi\} \left\{ \frac{3^4}{2 \ln 3} - \frac{1}{2 \ln 3} \right\} \left\{ = \frac{40\pi}{\ln 3} \right\}$	dM1		
$V_{\text{cone}} = \frac{1}{3} \pi (9)^2 \left( \frac{1}{\ln 3} \right) \left\{ = \frac{27\pi}{\ln 3} \right\}$ <span style="float: right;"><math>V_{\text{cone}} = \frac{1}{3} \pi (9)^2 (2 - \text{their (a)})</math>. See notes.</span>	B1ft		
$\left\{ \text{Vol}(S) = \frac{40\pi}{\ln 3} - \frac{27\pi}{\ln 3} \right\} = \frac{13\pi}{\ln 3}$ <span style="float: right;"><math>\frac{13\pi}{\ln 3}</math> or <math>\frac{26\pi}{\ln 9}</math> or <math>\frac{26\pi}{2 \ln 3}</math> etc., isw</span>	A1 o.e.		
	{Eg: $p = 13\pi$ , $q = \ln 3$ }	<b>[6]</b>	
(b)	<b>Alternative Method 1: Use of a substitution</b>	<b>10</b>	
	$V = \pi \int (3^x)^2 \{dx\}$	B1 o.e.	
	$\left\{ u = 3^x \Rightarrow \frac{du}{dx} = 3^x \ln 3 = u \ln 3 \right\} V = \{\pi\} \int \frac{u^2}{u \ln 3} \{du\} = \{\pi\} \int \frac{u}{\ln 3} \{du\}$		
	$= \{\pi\} \left\{ \frac{u^2}{2 \ln 3} \right\}$ <span style="float: right;"><math>(3^x)^2 \rightarrow \frac{u^2}{\pm \alpha (\ln 3)}</math> or <math>\pm \alpha (\ln 3) u^2</math>, where <math>u = 3^x</math></span>	M1	
		$(3^x)^2 \rightarrow \frac{u^2}{2 (\ln 3)}$ , where $u = 3^x$	A1
	$\left\{ V = \pi \int_0^2 (3^x)^2 dx = \{\pi\} \left[ \frac{u^2}{2 \ln 3} \right]_1^9 \right\} = \{\pi\} \left\{ \frac{9^2}{2 \ln 3} - \frac{1}{2 \ln 3} \right\} \left\{ = \frac{40\pi}{\ln 3} \right\}$ <span style="float: right;">Substitutes limits of 9 and 1 in <math>u</math> (or 2 and 0 in <math>x</math>) and subtracts the correct way round.</span>	dM1	
	<i>then apply the main scheme.</i>		

		<b>Question 8 Notes</b>
8. (a)	<b>B1</b>	$\frac{dy}{dx} = 3^x \ln 3$ or $\ln 3(e^{x \ln 3})$ or $y \ln 3$ . Can be implied by later working.
	<b>M1</b>	Substitutes either $x = 2$ or $y = 9$ into their $\frac{dy}{dx}$ which is a function of $x$ or $y$ to find $m_T$ <b>and</b> <ul style="list-style-type: none"> <li>• <b>either</b> applies <math>y - 9 = (\text{their } m_T)(x - 2)</math>, where <math>m_T</math> is a numerical value.</li> <li>• <b>or</b> applies <math>y = (\text{their } m_T)x + \text{their } c</math>, where <math>m_T</math> is a numerical value and <math>c</math> is found by solving <math>9 = (\text{their } m_T)(2) + c</math></li> </ul>
	<b>Note</b>	The first M1 mark can be implied from later working.
	<b>M1</b>	Sets $y = 0$ in their <b>tangent</b> equation, where $m_T$ is a numerical value, (seen or implied) and progresses to $x = \dots$
	<b>A1</b>	An exact value of $2 - \frac{1}{\ln 3}$ or $\frac{2 \ln 3 - 1}{\ln 3}$ or $\frac{\ln 9 - 1}{\ln 3}$ by a correct solution only.
	<b>Note</b>	Allow A1 for $2 - \frac{\lambda}{\lambda \ln 3}$ or $\frac{\lambda(2 \ln 3 - 1)}{\lambda \ln 3}$ or $\frac{\lambda(\ln 9 - 1)}{\lambda \ln 3}$ or $2 - \frac{\lambda}{\lambda \ln 3}$ , where $\lambda$ is an integer, and ignore subsequent working.
	<b>Note</b>	Using a changed gradient (i.e. applying $\frac{-1}{\text{their } \frac{dy}{dx}}$ or $\frac{1}{\text{their } \frac{dy}{dx}}$ ) is M0 M0 in part (a).
	<b>Note</b>	Candidates who invent a value for $m_T$ (which bears no resemblance to their gradient function) <b>cannot</b> gain the 1 <sup>st</sup> M1 and 2 <sup>nd</sup> M1 mark in part (a).
	<b>Note</b>	A decimal answer of 1.089760773... (without a correct <b>exact</b> answer) is A0.
	8. (b)	<b>B1</b>
<b>Note</b>		<b>Eg:</b> Allow B1 for $\pi \int (3^x)^2 \{dx\}$ or $\pi \int 3^{2x} \{dx\}$ or $\pi \int 9^x \{dx\}$ or $\pi \int (e^{x \ln 3})^2 \{dx\}$ or $\pi \int (e^{2x \ln 3}) \{dx\}$ or $\pi \int e^{x \ln 9} \{dx\}$ with or without $dx$
<b>M1</b>		Either $3^{2x} \rightarrow \frac{3^{2x}}{\pm \alpha (\ln 3)}$ or $\pm \alpha (\ln 3) 3^{2x}$ or $9^x \rightarrow \frac{9^x}{\pm \alpha (\ln 9)}$ or $\pm \alpha (\ln 9) 9^x$ $e^{2x \ln 3} \rightarrow \frac{e^{2x \ln 3}}{\pm \alpha (\ln 3)}$ or $\pm \alpha (\ln 3) e^{2x \ln 3}$ or $e^{x \ln 9} \rightarrow \frac{e^{x \ln 9}}{\pm \alpha (\ln 9)}$ or $\pm \alpha (\ln 9) e^{x \ln 9}$ , etc where $\alpha \in \mathbb{R}$
<b>Note</b>		$3^{2x} \rightarrow \frac{3^{2x+1}}{\pm \alpha (\ln 3)}$ or $9^x \rightarrow \frac{9^{x+1}}{\pm \alpha (\ln 3)}$ are allowed for M1
<b>Note</b>		$3^{2x} \rightarrow \frac{3^{2x+1}}{2x+1}$ or $9^x \rightarrow \frac{9^{x+1}}{x+1}$ are both M0
<b>Note</b>		M1 can be given for $9^{2x} \rightarrow \frac{9^{2x}}{\pm \alpha (\ln 9)}$ or $\pm \alpha (\ln 9) 9^{2x}$
<b>A1</b>		Correct integration of $3^{2x}$ . Eg: $3^{2x} \rightarrow \frac{3^{2x}}{2 \ln 3}$ or $\frac{3^{2x}}{\ln 9}$ or $9^x \rightarrow \frac{9^x}{\ln 9}$ or $e^{2x \ln 3} \rightarrow \frac{1}{2 \ln 3} (e^{2x \ln 3})$
<b>dM1</b>	<b>dependent on the previous method mark being awarded.</b>	
<b>Note</b>	Attempts to apply $x = 2$ and $x = 0$ to integrated expression and subtracts the correct way round. Evidence of a proper consideration of the limit of 0 is needed for M1. So subtracting 0 is M0.	

	<p><b>dM1</b> <b>dependent on the previous method mark being awarded.</b> Attempts to apply <math>x = 2</math> and <math>x = 0</math> to integrated expression and subtracts the correct way round.</p> <p><b>Note</b> Evidence of a proper consideration of the limit of 0 is needed for M1. So subtracting 0 is M0.</p> <p><b>B1ft</b> <math>V_{\text{cone}} = \frac{1}{3}\pi(9)^2(2 - \text{their answer to part (a)})</math>.</p> <p>Sight of <math>\frac{27\pi}{\ln 3}</math> implies the B1 mark.</p> <p><b>Note</b> Alternatively they can apply the volume formula to the line segment. They need to achieve the result highlighted by **** on either page 29 or page 30 in order to obtain the B1ft mark.</p>
	<p><b>A1</b> <math>\frac{13\pi}{\ln 3}</math> or <math>\frac{26\pi}{\ln 9}</math> or <math>\frac{26\pi}{2\ln 3}</math>, etc., where their answer is in the form <math>\frac{p}{q}</math></p> <p><b>Note</b> The <math>\pi</math> in the volume formula is only needed for the 1<sup>st</sup> B1 mark and the final A1 mark.</p> <p><b>Note</b> A decimal answer of 37.17481128... (without a correct <b>exact</b> answer) is A0.</p> <p><b>Note</b> A candidate who applies <math>\int 3^x dx</math> will either get B0 M0 A0 M0 <b>B0 A0</b> or B0 M0 A0 M0 <b>B1 A0</b></p> <p><b>Note</b> <math>\pi \int 3^{x^2} dx</math> unless recovered is B0.</p> <p><b>Note</b> <b><u>Be careful! A correct answer may follow from incorrect working</u></b></p> <p><math display="block">V = \pi \int_0^2 3^{x^2} dx - \frac{1}{3}\pi(9)^2 \left( \frac{1}{\ln 3} \right) = \pi \left[ \frac{3^{x^2}}{2\ln 3} \right]_0^2 - \frac{27\pi}{\ln 3} = \frac{\pi 3^4}{2\ln 3} - \frac{\pi}{2\ln 3} - \frac{27\pi}{\ln 3} = \frac{13\pi}{\ln 3}</math></p> <p>would score B0 M0 A0 dM0 M1 A0.</p>
8. (b)	<p><b>2<sup>nd</sup> B1ft mark for finding the Volume of a Cone</b></p> <p><math display="block">V_{\text{cone}} = \pi \int_{2 - \frac{1}{\ln 3}}^2 (9x \ln 3 - 18 \ln 3 + 9)^2 dx</math></p> <p><math display="block">= \pi \left[ \frac{(9x \ln 3 - 18 \ln 3 + 9)^3}{27 \ln 3} \right]_{2 - \frac{1}{\ln 3} \text{ or their part (a) answer}}^2</math> ****</p> <p><math display="block">= \pi \left( \left( \frac{(18 \ln 3 - 18 \ln 3 + 9)^3}{27 \ln 3} \right) - \left( \frac{\left( 9 \left( 2 - \frac{1}{\ln 3} \right) \ln 3 - 18 \ln 3 + 9 \right)^3}{27 \ln 3} \right) \right)</math></p> <p><math display="block">= \pi \left( \left( \frac{729}{27 \ln 3} \right) - \left( \frac{(18 \ln 3 - 9 - 18 \ln 3 + 9)^3}{27 \ln 3} \right) \right)</math></p> <p><math display="block">= \frac{27\pi}{\ln 3}</math></p> <p>Award B1ft here where their lower limit is <math>2 - \frac{1}{\ln 3}</math> or their part (a) answer.</p>

8. (b)

**2<sup>nd</sup> B1ft mark for finding the Volume of a Cone**

**Alternative method 2:**

$$\begin{aligned}
 V_{\text{cone}} &= \pi \int_{2-\frac{1}{\ln 3}}^2 (9x \ln 3 - 18 \ln 3 + 9)^2 dx \\
 &= \pi \int_{2-\frac{1}{\ln 3}}^2 (81x^2 (\ln 3)^2 - 324x (\ln 3)^2 + 162x \ln 3 - 324 \ln 3 + 324 (\ln 3)^2 + 81) dx \\
 &= \pi \left[ 27x^3 (\ln 3)^2 - 162x^2 (\ln 3)^2 + 81x^2 \ln 3 - 324x \ln 3 + 324x (\ln 3)^2 + 81x \right]_{2-\frac{1}{\ln 3}}^2
 \end{aligned}$$

Award B1ft here where their lower limit is  $2 - \frac{1}{\ln 3}$  or their part (a) answer.

\*\*\*\*

$$\begin{aligned}
 &= \pi \left( \left( 216 (\ln 3)^2 - 648 (\ln 3)^2 + 324 \ln 3 - 648 \ln 3 + 648 (\ln 3)^2 + 162 \right) \right. \\
 &\quad \left. - \left( 27 \left( 2 - \frac{1}{\ln 3} \right)^3 (\ln 3)^2 - 162 \left( 2 - \frac{1}{\ln 3} \right)^2 (\ln 3)^2 + 81 \left( 2 - \frac{1}{\ln 3} \right)^2 \ln 3 \right) \right. \\
 &\quad \left. - \left( -324 \left( 2 - \frac{1}{\ln 3} \right) \ln 3 + 324 \left( 2 - \frac{1}{\ln 3} \right) (\ln 3)^2 + 81 \left( 2 - \frac{1}{\ln 3} \right) \right) \right) \\
 &= \pi \left( \left( 216 (\ln 3)^2 - 324 \ln 3 + 162 \right) - \left( 27 \left( 8 - \frac{12}{\ln 3} + \frac{6}{(\ln 3)^2} - \frac{1}{(\ln 3)^3} \right) (\ln 3)^2 - 162 \left( 4 - \frac{4}{\ln 3} + \frac{1}{(\ln 3)^2} \right) (\ln 3)^2 \right) \right. \\
 &\quad \left. + 81 \left( 4 - \frac{4}{\ln 3} + \frac{1}{(\ln 3)^2} \right) \ln 3 - 324 \left( 2 - \frac{1}{\ln 3} \right) \ln 3 \right. \\
 &\quad \left. + 324 \left( 2 - \frac{1}{\ln 3} \right) (\ln 3)^2 + 81 \left( 2 - \frac{1}{\ln 3} \right) \right) \\
 &= \pi \left( \left( 216 (\ln 3)^2 - 324 \ln 3 + 162 \right) - \left( 216 (\ln 3)^2 - 324 \ln 3 + 162 - \frac{27}{\ln 3} - 648 (\ln 3)^2 + 648 \ln 3 - 162 \right) \right. \\
 &\quad \left. + 324 \ln 3 - 324 + \frac{81}{\ln 3} - 648 \ln 3 + 324 \right. \\
 &\quad \left. + 648 (\ln 3)^2 - 324 \ln 3 + 162 - \frac{81}{\ln 3} \right) \\
 &= \pi \left( \left( 216 (\ln 3)^2 - 324 \ln 3 + 162 \right) - \left( 216 (\ln 3)^2 - 324 \ln 3 + 162 - \frac{27}{\ln 3} \right) \right) \\
 &= \frac{27\pi}{\ln 3}
 \end{aligned}$$

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