## General Certificate of Secondary Education 2019

# Mathematics 

M4
Calculator Paper
Higher Tier
[GMC41]

TUESDAY 21 MAY, 9.15am-11.15am

## MARK <br> SCHEME

## GCSE MATHEMATICS

## Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right hand column and they are prefixed by the letters $\mathbf{M}, \mathbf{A}$ and $\mathbf{M A}$ as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.
A indicates marks for accurate working, whether in calculation, reading from tables, graphs or answers. Accuracy marks may depend on preceding M (method) marks, hence M0 A1 cannot be awarded, i.e. where the method is not correct no marks can be given.

MA indicates marks for combined method and accurate working.
A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

## Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of following through their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:
(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;
(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

## General Marking Advice

(i) If the correct answer is seen in the body of the script and the answer given in the answer line is clearly a transcription error, full marks should be awarded.
(ii) If the answer is missing, but the correct answer is seen in the body of the script, full marks should be awarded.
(iii) If the correct answer is seen in working but a completely different answer is seen in the answer space, then some marks will be awarded depending on the severity of the error.
(iv) Work crossed out but not replaced should be marked.
(v) In general, if two or more methods are offered, mark only the method that leads to the answer on the answer line, if two (or more) answers are offered (with no solution offered on the answer line), mark the poorest answer.
(vi) For methods not provided for in the mark scheme, give as far as possible equivalent marks for equivalent work.
(vii) Where a follow through mark is indicated on the mark scheme for a particular part question, the marker must ensure that you refer back to the answer of the previous part of the question.
(viii) Unless the question asks for an answer to a specific degree of accuracy, always mark at the greatest number of significant figures seen, e.g. the answer in the mark scheme is 4.65 and the candidtae then correctly round to 4.7 or 5 on the answer line. Allow full marks for 4.65 seen in the working.
(ix) Anything in the mark scheme which is in brackets (...) is not required for the mark to be earned, but if present it must be correct.
(x) For any question, the range of answers given in the mark scheme is inclusive.

| 1 | $\begin{aligned} & 2.5 \times 7+7.5 \times 8+12.5 \times 5+17.5 \times 5+22.5 \times 4+27.5 \times 1(=345) \\ & 345 / 30 \end{aligned}$ | M1 A1 <br> MA1 | AVAILABLE MARKS |
| :---: | :---: | :---: | :---: |
|  | $=11.5$ | A1 | 4 |
| 2 | $\begin{aligned} & 8 x-12-2 x+10 \\ & =6 x-2 \end{aligned}$ | MA1 <br> MA1 | 2 |
| 3 | $\begin{aligned} & 200=2 \times 2 \times 2 \times 5 \times 5 \\ & 2^{3} \times 5^{2} \end{aligned}$ | $\begin{array}{r} \mathrm{M} 1 \mathrm{~A} 1 \\ \mathrm{~A} 1 \end{array}$ | 3 |
| 4 | $\begin{aligned} & 2 x+10+2 x+x+20=180 \\ & 5 x=150 \\ & x=30 \\ & \text { Smallest angle }=50 \end{aligned}$ | $\begin{array}{r} \text { M1 } \\ \text { MA1 } \\ \text { MA1 } \\ \text { A1 } \end{array}$ | 4 |
| 5 | Diagram correctly set up $\begin{aligned} & 4^{2}+x^{2}=12^{2} \\ & x^{2}=128 \\ & x=11.3 \end{aligned}$ <br> Perimeter $=4+12+11.3=27.3$ | $\begin{array}{r} \text { M1 } \\ \text { MA1 } \\ \text { MA1 } \\ \text { MA1 } \\ \text { MA1 } \end{array}$ | 5 |
| 6 | (a) $20 \%$ increase, e.g. $120 \%$ of $150=180$ <br> $20 \%$ decrease, e.g. $80 \%$ of $180=144-$ decrease | M1 A1 <br> MA1 |  |
|  | (b) $\%$ decrease $=4 \%$ | M1 A1 |  |
|  | (c) $(20 \%$ decrease $)=0.8 \times 1.2(20 \%$ increase $)=$ same (or similar explanation/calculation to justify answer) | M1 A1 | 7 |
| 7 | $\begin{aligned} & \text { SA of sphere }=4 \times \pi \times 6^{2}=452 \\ & \text { SA hemisphere }=226 \\ & \text { base }=\pi \times 6^{2}=113 \\ & \text { total }=339 \end{aligned}$ Martha is correct | MA1 <br> MA1 <br> MA1 <br> MA1 | 4 |
| 8 | $\begin{aligned} & \text { distance travelled in } 1 \text { minute }=720 \mathrm{~m} \\ & \sin 20=x / 720 \\ & x=720 \sin 20 \\ & x=246.25 \mathrm{~m} \end{aligned}$ | MA1 <br> MA1 <br> MA1 <br> A1 | 4 |
| 9 | $\begin{aligned} & (x-4)(x+3)=0 \\ & x=4 \quad x=-3 \end{aligned}$ | $\begin{array}{r} \mathrm{M} 1 \mathrm{~A} 1 \\ \mathrm{~A} 1 \end{array}$ | 3 |

$10 \quad 2(a-1)+(a+1)=12$
$2 a-2+a+1=12$
$a=4 \frac{1}{3}$
$11 y=3 x+c(c=$ any numerical value, $\mathrm{c} \neq 5)$
$12 \quad 107.5 \%=29455$
$100 \%=29455 \div 107.5 \times 100$
$=£ 27400$

## Alternative Solution

$100 \%=29455 \div 1.075$
$=£ 27400$

13 (a) Cumulative frequency graph and scale
(b) 160 - (reading from 55 on their graph)
(correct reading approx. $160-72=88$ ) Allow A1 for reading at 55

14 Year 8: $\frac{126}{756} \times 50=8$ or 9

15 (a) $5.75-3.25=2.5$
(b) $\frac{5.85}{3.15}=1 . \dot{8} 5714 \dot{2}$

16 (a) perimeter of rectangle $=16+4 x$
Side of square $=4+x$
(b) (i) $(4+x)^{2}=16 x+4$
$16+8 x+x^{2}=16 x+4$
$x^{2}-8 x+12=0$
(ii) $(x-2)(x-6)=0$ $x=2,6$
(b) $16+8 x+x^{2}=16 x+4$ MA1
$17 l_{1}: \mathrm{m}=\frac{8-(-4)}{2-(-1)}=4$
$l_{2}: \mathrm{m}=-\frac{1}{4}$
$y=-\frac{1}{4} x+c$ or $y-1=-\frac{1}{4}(x-1)$
$1=-\frac{1}{4}(1)+c$
$c=\frac{5}{4}$
$y=-\frac{1}{4} x+\frac{5}{4}$
$18 \quad \operatorname{Cos} \mathrm{AOC}=\frac{5}{13}$
$\mathrm{AOC}=67.4^{\circ}$
Area of sector $\mathrm{AOB}=\frac{67.4}{360} \times \pi \times 13^{2}=99.4$
M1 A1
$\mathrm{AC}^{2}=13^{2}-5^{2}$
$\mathrm{AC}=12$
Area of triangle $\mathrm{OAC}=\frac{1}{2} \times 5 \times 12=30$
M1 A1

Shaded Area $=99.4-30=69.4 \mathrm{~cm}^{2}$
$19 \frac{4(x+4)-3(x+3)}{(x+3)(x+4)}=1$
$\frac{x+7}{x^{2}+7 x+12}=1$
$x+7=x^{2}+7 x+12$
$x^{2}+6 x+5=0$
$(x+1)(x+5)=0 \quad$ MA1
$x=-1,-5$

20 (a) 30, 15

| $0-10$ bar, freq. density of 1.0 | MA1 |
| :--- | :--- |
| $10-20$ bar, freq. density of 2.5 | MA1 |
| $90-100$ bar, freq. density of 1.5 | MA1 |

$100-120$ bar, freq. density of 0.5 MA1
(b) Median $=40+\frac{50}{80} \times 20 \quad$ MA1

$$
=52.5 \text { minutes } \quad \mathrm{A} 1
$$

21 Angle $\mathrm{PSQ}=41^{\circ}$ (alternate segment theorem)
Angle $\mathrm{SPQ}=82^{\circ}(180-57-41$ triangle $)$
(or Angle SPT $=57^{\circ}$ (alternate segment theorem)
(and Angle $\mathrm{SPQ}=82^{\circ}$ (straight line)
Angle $\mathrm{SRQ}=98^{\circ}$ (opposite angle in cyclic quadrilateral) MA1
Angle $\mathrm{SQR}=41^{\circ}$ (isosceles triangle) MA1
So SP parallel to RQ as alternate angles PSQ and SQR are equal

22 (a) $(2 a-b)(a+4 b)$
(b) $\frac{(x+1)(x-4)+(3 x-4)(2 x-1)}{(2 x-1)(x-4)} \times \frac{2 x-1}{x}$
$\frac{x^{2}-3 x-4+6 x^{2}-11 x+4}{(2 x-1)(x-4)} \times \frac{(2 x-1)}{x}$
$\frac{7 x^{2}-14 x}{(x-4)} \times \frac{1}{x}$
$\frac{7(x-2)}{x-4}$ or $\frac{7 x-14}{x-4}$

## Total

