

## Algebraic proof

Q1. Prove that  $(3n + 1)^2 - (3n - 1)^2$  is a multiple of 4, for all positive integer values of  $n$ .

Q2. Prove that the difference between the squares of consecutive odd numbers is a multiple of 8

Q3. Prove, using algebra, that the sum of two consecutive whole numbers is always an odd number.

Q4. Here are the first 4 lines of a number pattern.

$n$  is the first number in the  $n$ th line of the number pattern.

Show that the above number pattern is true for the four consecutive integers  $n$ ,  $(n + 1)$ ,  $(n + 2)$  and  $(n + 3)$

Q5.  $n$  and  $a$  are integers.

Explain why  $(n^2 - a^2) - (n - a)^2$  is always an even integer.

Q6. In some questions, a numerical example is used to give you a clue which will help you to write down an algebraic proof.

**a** Choose any odd number and any even number. Add these together. Is the result odd or even?

Does this always work for any odd number and even number you choose?

**b** Let any odd number be represented by  $2n + 1$ . Let any even number be represented by  $2m$ , where  $m$  and  $n$  are integers. Prove that the sum of an odd number and an even number always gives an odd number.

Q7. Prove the following results.

**a** the sum of two even numbers is even

**b** the product of two even numbers is even

**c** the product of an odd number and an even number is even

**d** the product of two odd numbers is odd

**e** the sum of four consecutive numbers is always even

**f** half the sum of four consecutive numbers is always odd



Q8. Speed Cabs charges 45 pence per kilometre for each journey. Evans Taxis has a fixed charge of 90p plus 30p per kilometre.

a i Verify that Speed Cabs is cheaper for a journey of 5 km.

ii Verify that Evans Taxis is cheaper for a journey of 7 km.

b Show clearly why both companies charge the same for a journey of 6 km.

c Show that if Speed Cabs charges  $a$  pence per kilometre, and Evans Taxis has a fixed charge of  $\pounds b$  plus a charge of  $c$  pence per kilometre, both companies charge the same for a journey of kilometres.

Q9. You are given that:

$$(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$$

a Verify that this result is true for  $a = 3$  and  $b = 4$ .

b Show that the LHS is the same as the RHS.

c Prove that the LHS can be simplified to the RHS.

Prove that  $(a + b)^2 - (a - b)^2 = 4ab$ .

$100b(a - c)$

Q10. Two integers have a difference of 3.

The difference between the squares of the two integers is three times the sum of the integers.

For example,  $13 - 10 = 3$ ,  $13^2 - 10^2 = 169 - 100 = 69$  and  $3 \times (13 + 10) = 3 \times 23 = 69$  Prove this result algebraically.

Q11. The difference between the squares of two consecutive even numbers is twice the sum of the numbers.

For example  $8^2 - 6^2 = 28$

$2 \times (8 + 6) = 28$

Prove this result algebraically.

Q12. Two integers,  $a$  and  $b$ , are combined using the operation  $\square$  in the following way.

$$a \ast b = a^2 + a - 4b - b^2$$

(a) Find all solutions to the equation  $x \ast 2 = 0$

.....

.....

.....

.....

.....

Answer .....

(b) If  $a$  is 4 greater than  $b$ , prove that  $a \ast b$  is always a multiple of 5.

.....

.....



Q13. Prove that the sum of five consecutive integers must end in a 0 or a 5

Here is an identity.

$$(2x + a)(x + 3) \equiv 2x^2 + 4ax + b$$

a and b are numbers.

Work out b.

Q14. The sum of the squares of two consecutive integers is one greater than twice the product of the integers.

For example  $9^2 + 10^2 = 81 + 100$  and  $2 \times 9 \times 10 = 180$   
 $= 181$

Prove this result algebraically.

Q15. Mathematically similar and expressing one quantity in terms of another (radius in terms of height)

Q16. Prove that the sum of two consecutive odd numbers is always the multiple of 4. Prove it algebraically.

Q17. Prove that the sum of two consecutive triangular numbers is always a square number.