Proportionality

Q1. The distance covered by a train is directly proportional to the time taken. The train travels 105 miles in 3 hours.
a What distance will the train cover in 5 hours? _____

b What time will it take for the train to cover 280 miles? _____

Q2. The cost of fuel delivered to your door is directly proportional to the weight received. When 250 kg is delivered, it costs £47.50.
a How much will it cost to have 350 kg delivered? _____

b How much would be delivered if the cost were £33.25? _____

Q3. The number of children who can play safely in a playground is directly proportional to the area of the playground. A playground with an area of 210 m^2 is safe for 60 children.
a How many children can safely play in a playground of area 154 m^2?

_____

b A playground has 24 children. What is the smallest playground area in which they could safely play?

_____

Q4. In each case, first find k, the constant of proportionality, and then the formula connecting the variables.

i) T is directly proportional to x^2. If $T = 36$ when $x = 3$, find the following.

a T when $x = 5$ _____ **b** x when $T = 400$ _____

ii) W is directly proportional to M^2. If $W = 12$ when $M = 2$, find the following.

a W when $M = 3$ _____ **b** M when $W = 75$ _____

iii) E varies directly with $\sqrt{C}$. If $E = 40$ when $C = 25$, find the following.

a E when $C = 49$ _____ **b** C when $E = 10.4$ _____

iv) X is directly proportional to $\sqrt{Y}$. If $X = 128$ when $Y = 16$, find the following.

a X when $Y = 36$ _____ **b** Y when $X = 48$ _____

v) P is directly proportional to f^3. If $P = 400$ when $f = 10$, find the following.

a P when $f = 4$ _____ **b** f when $P = 50$ _____

Q5. The cost of serving tea and biscuits varies directly with the square root of the number of people at the buffet. It costs £25 to serve tea and biscuits to 100 people.

a How much will it cost to serve tea and biscuits to 400 people? _____

b For a cost of £37.50, how many could be served tea and biscuits? _____

Q6. In an experiment, the temperature, in °C, varied directly with the square of the pressure, in atmospheres. The temperature was 20 °C when the pressure was 5 atm.
a What will the temperature be at 2 atm? _____

b What will the pressure be at 80 °C? _____

Q7. The weight, in grams, of ball bearings varies directly with the cube of the radius measured in millimetres. A ball bearing of radius 4 mm has a weight of 115.2 g.
a What will a ball bearing of radius 6 mm weigh? _____

b A ball bearing has a weight of 48.6 g. What is its radius? _____

Q8. The energy, in J, of a particle varies directly with the square of its speed in m/s. A particle moving at 20 m/s has 50 J of energy.
a How much energy has a particle moving at 4 m/s? _____

b At what speed is a particle moving if it has 200 J of energy? _____

Q9. The cost, in £, of a trip varies directly with the square root of the number of miles travelled. The cost of a 100-mile trip is £35.
a What is the cost of a 500-mile trip (to the nearest £1)? _____

b What is the distance of a trip costing £70? _____

Q10. In each case, first find the formula connecting the variables.

i) T is inversely proportional to m. If $T = 6$ when $m = 2$, find the following.
a T when $m = 4$ _____ **b** m when $T = 4.8$ _____

ii) W is inversely proportional to x. If $W = 5$ when $x = 12$, find the following.
a W when $x = 3$ _____ **b** x when $W = 10$ _____

iii) Q varies inversely with $(5 - t)$. If $Q = 8$ when $t = 3$, find the following.
a Q when $t = 10$ _____ **b** t when $Q = 16$ _____

iv) M varies inversely with $t\,2$. If $M = 9$ when $t = 2$, find the following.
a M when $t = 3$ _____ **b** t when $M = 1.44$ _____

v) W is inversely proportional to $\sqrt{T}$. If $W = 6$ when $T = 16$, find the following.
a W when $T = 25$ _____ **b** T when $W = 2.4$ _____

Q11. The grant available to a section of society was inversely proportional to the number of people needing the grant. When 30 people needed a grant, they received £60 each.

a What would the grant have been if 120 people had needed one? _____

b If the grant had been £50 each,
how many people would have received it? _____

Q12. While doing underwater tests in one part of an ocean, a team of scientists noticed that the temperature in °C was inversely proportional to the depth in kilometres. When the temperature was 6 °C, the scientists were at a depth of 4 km.

a What would the temperature have been at a depth of 8 km? _____

b To what depth would they have had to go to
find the temperature at 2 °C? _____

Q13. A new engine was being tested, but it had serious problems. The distance it went, in km, without breaking down was inversely proportional to the square of its speed in m/s. When the speed was 12 m/s, the engine lasted 3 km.

a Find the distance covered before a breakdown,
when the speed is 15 m/s. _____

b On one test, the engine broke down after 6.75 km.
What was the speed? _____

Q14. In a balloon it was noticed that the pressure, in atmospheres, was inversely proportional to the square root of the height, in metres. When the balloon was at a height of 25 m, the pressure was 1.44 atm.

a What was the pressure at a height of 9 m? _____

b What would the height have been if the pressure was 0.72 atm? _____

Q15. The amount of waste which a firm produces, measured in tonnes per hour, is inversely proportional to the square root of the size of the filter beds, measured in m^2. At the moment, the firm produces 1.25 tonnes per hour of waste, with filter beds of size 0.16 m^2.

a The filter beds used to be only 0.01 m^2.
How much waste did the firm produce then? _____

b How much waste could be produced
if the filter beds were 0.75 m^2? _____