

Recurring Decimal

Non-Terminating Recurring Decimal as a Fraction

If a decimal goes on forever (non-terminating) but repeats, then it is rational i.e. it can be written as a fraction. The obvious example is $0.33333333\ldots$ which is $\frac{1}{3}$

Some numbers are not quite so obvious but can still be written as fractions

e.g. 1. Write $0.\dot{4}$ as a fraction

Let $x = 0.444444\ldots$ so $10x = 4.444444\ldots$

$$10x - x = 4 = 9x$$

$$\text{If } 9x = 4 \text{ then } x = \frac{4}{9} \text{ but as } x = 0.\dot{4} \quad 0.\dot{4} = \frac{4}{9}$$

e.g.2. Write $3.\dot{4}$ as a fraction

We know that $0.\dot{4} = \frac{4}{9}$ so $3.\dot{4}$ must equal $3\frac{4}{9}$ or $\frac{31}{9}$

e.g.3. Write $0.\dot{2}\dot{3}$ as a fraction

$$0.\dot{2}\dot{3} = 0.232323\ldots$$

Let $x = 0.\dot{2}\dot{3}$ so $100x = 23.232323\ldots$

$$100x - x = 23.232323\ldots - 0.232323\ldots = 23 = 99x$$

$$\text{If } 99x = 23 \text{ then } x = \frac{23}{99} \text{ but } x = 0.232323\ldots$$

$$\text{So } 0.\dot{2}\dot{3} = \frac{23}{99}$$

After a while you can just recognise the answers *if* the recurring part of the decimal is next to the decimal point

$$0.\dot{1}5\dot{2} = \frac{152}{999} \quad 0.\dot{2}\dot{7} = \frac{27}{99} = \frac{3}{11} \text{ (in simplest form)}$$

However you must remember the 'long-winded' method as you may be asked to prove that, for example, $0.\dot{2}\dot{3} = \frac{23}{99}$

If the recurring part is not next to the decimal point then we have to split the recurring and non-recurring parts and use a fancy bit of division/multiplication to put the recurring part next to a decimal point. This is probably going to appear on the Higher paper.

e.g.4. Write $0.2\dot{5}$ as a fraction

$$\begin{aligned}
 0.2\dot{5} &= 0.2 + 0.0\dot{5} \\
 &= 0.2 + 0.\dot{5} \div 10 \\
 &= \frac{2}{10} + \frac{5}{9} \div 10 \\
 &= \frac{2}{10} + \frac{5}{90} \\
 &= \frac{18}{90} + \frac{5}{90} \\
 &= \frac{23}{90}
 \end{aligned}$$

Split into recurring and non-recurring parts

Move recurring part next to decimal point

Write as fractions

Perform the division

Find common denominator and re-write using equivalent fractions

Add up

e.g. 5. Write $0.41\dot{5}\dot{3}$ as a fraction

$$\begin{aligned}
 &= 0.41 + 0.00\dot{5}\dot{3} \\
 &= 0.41 + 0.\dot{5}\dot{3} \div 100 \\
 &= \frac{41}{100} + \frac{53}{99} \div 100 \\
 0.41\dot{5}\dot{3} &= \frac{41}{100} + \frac{53}{9900} \\
 &= \frac{4059}{9900} + \frac{53}{9900} \\
 &= \frac{4112}{9900} \\
 &= \frac{1028}{2475}
 \end{aligned}$$

You can check your answer by converting the fraction to a decimal by dividing and seeing if you get the starting value e.g. $\frac{1028}{2475} = 1028 \div 2475 = 0.415353....$

Exercise:

Convert the following decimal numbers into fraction.

- | | | | |
|-------------------------|-------|------------------------|-------|
| Q1) $0.333333.....$ | _____ | Q2) $0.25252525.....$ | _____ |
| Q3) $1.555555.....$ | _____ | Q4) $0.125125125.....$ | _____ |
| Q5) $0.23333333.....$ | _____ | Q6) $0.46666666.....$ | _____ |
| Q7) $3.4545454545.....$ | _____ | Q8) $0.25363636.....$ | _____ |
| Q9) $24.288888.....$ | _____ | Q10) $12.23151515...$ | _____ |