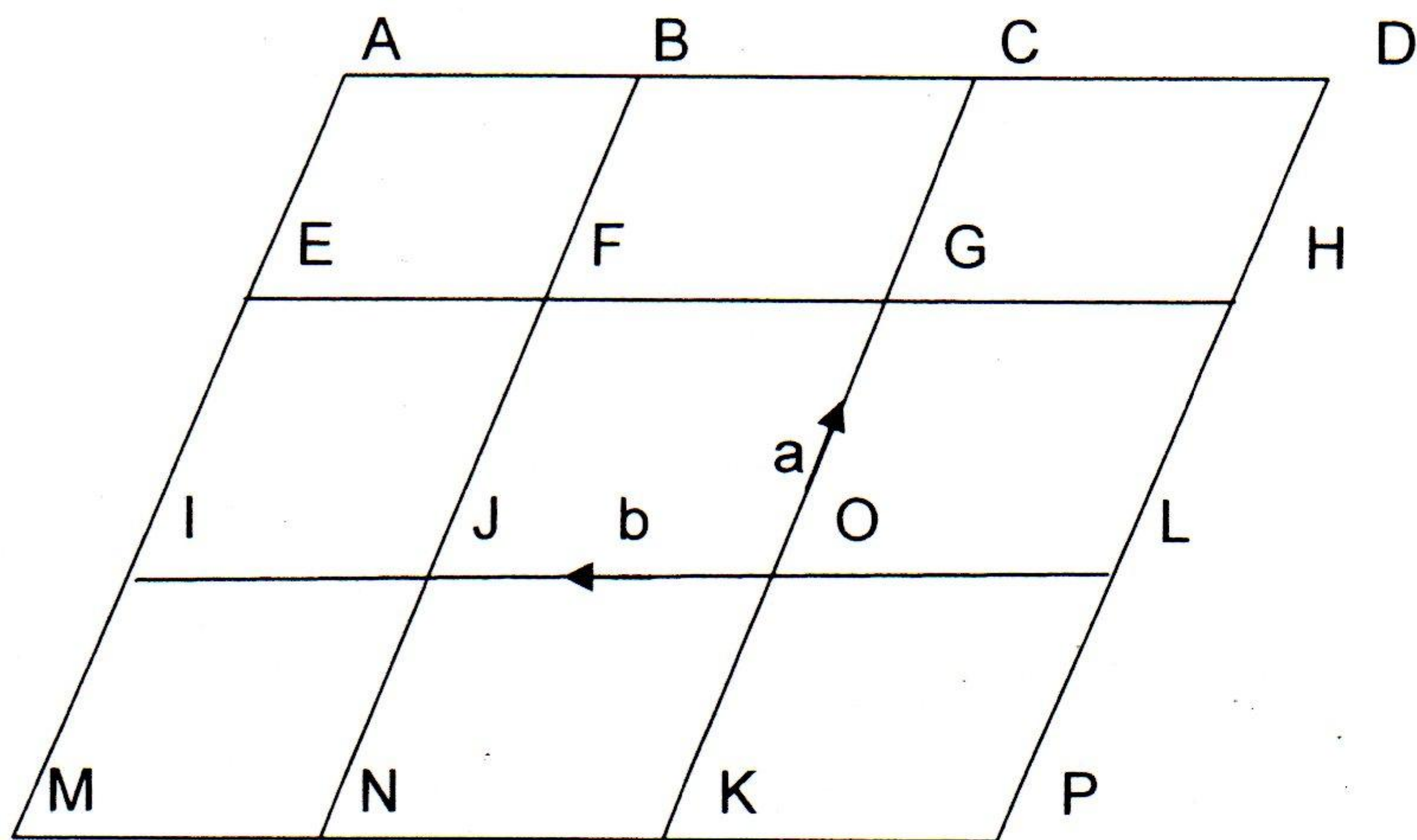


Vector



1) Here in the figure $\vec{OG} = \vec{a}$ and $\vec{OJ} = \vec{b}$

Find the following vectors

AB=	NB=	OP=	FD=	PA=
AM=	NH=	OD=	FP=	PJ=
AH=	NE=	OM=	FM=	PI=

2) Find the position vector of the following vectors.

If $\vec{OP} = (2, 3)$ $\vec{OQ} = (5, 8)$ $\vec{OR} = (-7, 8)$ $\vec{OS} = (0, -5)$
 $\vec{OA} = (7, 2)$ $\vec{OB} = (5, 7)$ $\vec{OC} = (5, -9)$ $\vec{OD} = (-3, -7)$

Find the following vectors.

AB=	QB=	CP=	SD=	AA=
AS=	QD=	CD=	DQ=	PO=
AR=	QA=	CR=	DP=	DO=

3) On a copy of this grid, mark on the points C to P to show the following.

a $OC = 2a - b$

b $OD = 2a + b$

c $OE = a - 2b$

d $OF = b - 2a$

e $OG = -a$

f $OH = -a - 2b$

g $OI = 2a - 2b$

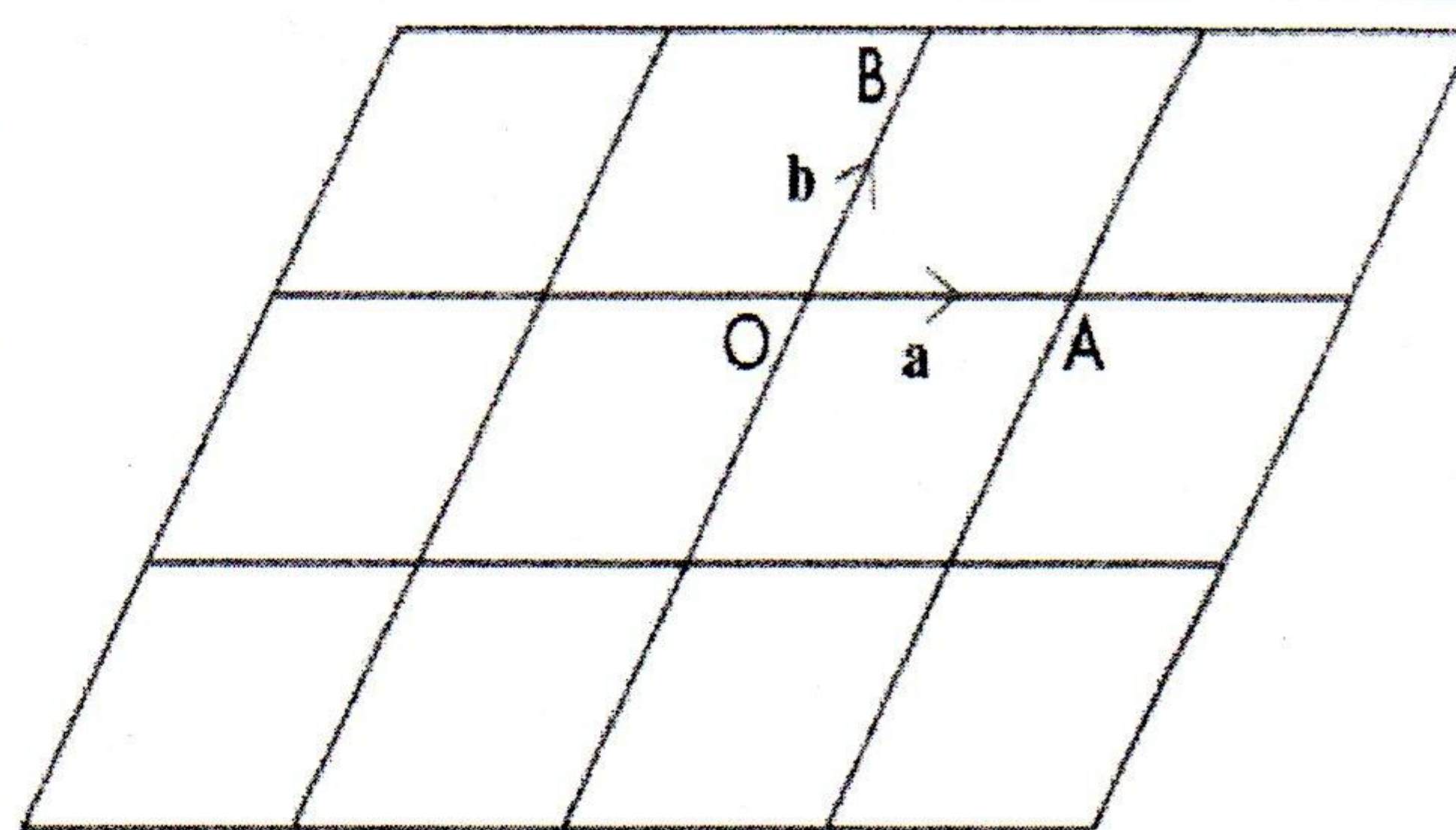
h $OJ = -a + b$

i $OK = -a - b$

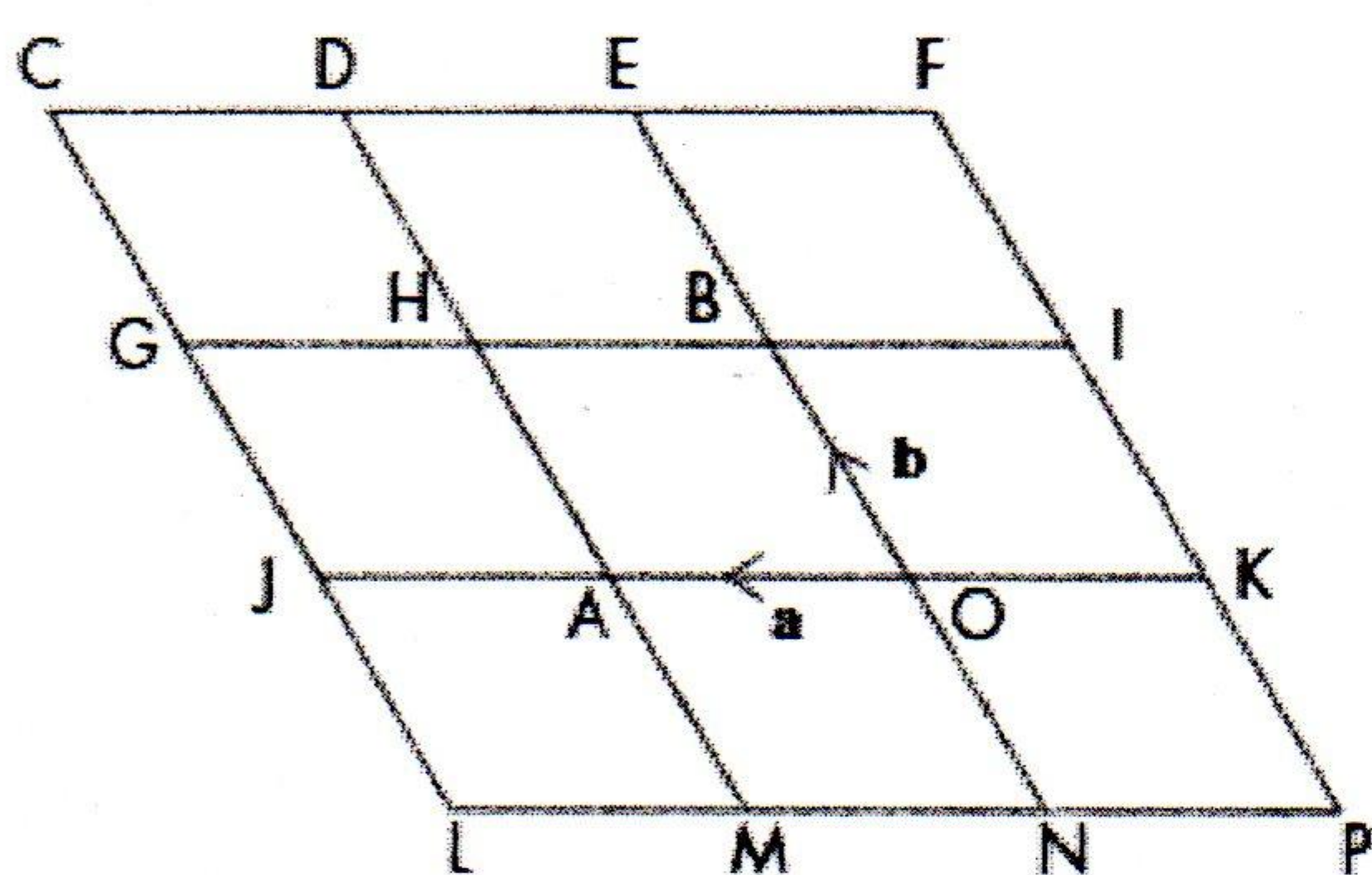
j $OM = -a - \frac{3}{2}b$

k $ON = -\frac{1}{2}a - 2b$

l $OP = \frac{3}{2}a - \frac{3}{2}b$



4) This grid shows the vectors
 $OA = a$ and $OB = b$.



a Name three vectors equivalent to $a + b$.

_____, _____, _____

b Name three vectors equivalent to $a - b$.

_____, _____, _____

c Name three vectors equivalent to $b - a$.

_____, _____, _____

d Name three vectors equivalent to $-a - b$.

_____, _____, _____

e Name three vectors equivalent to $2a - b$.

_____, _____, _____

f Name three vectors equivalent to $2b - a$.

_____, _____, _____

g For each of these, name one equivalent vector.

i $3a - b$

ii $2(a + b)$

iii $3a - 2b$

iv $3(a - b)$

v $3(b - a)$

vi $3(a + b)$

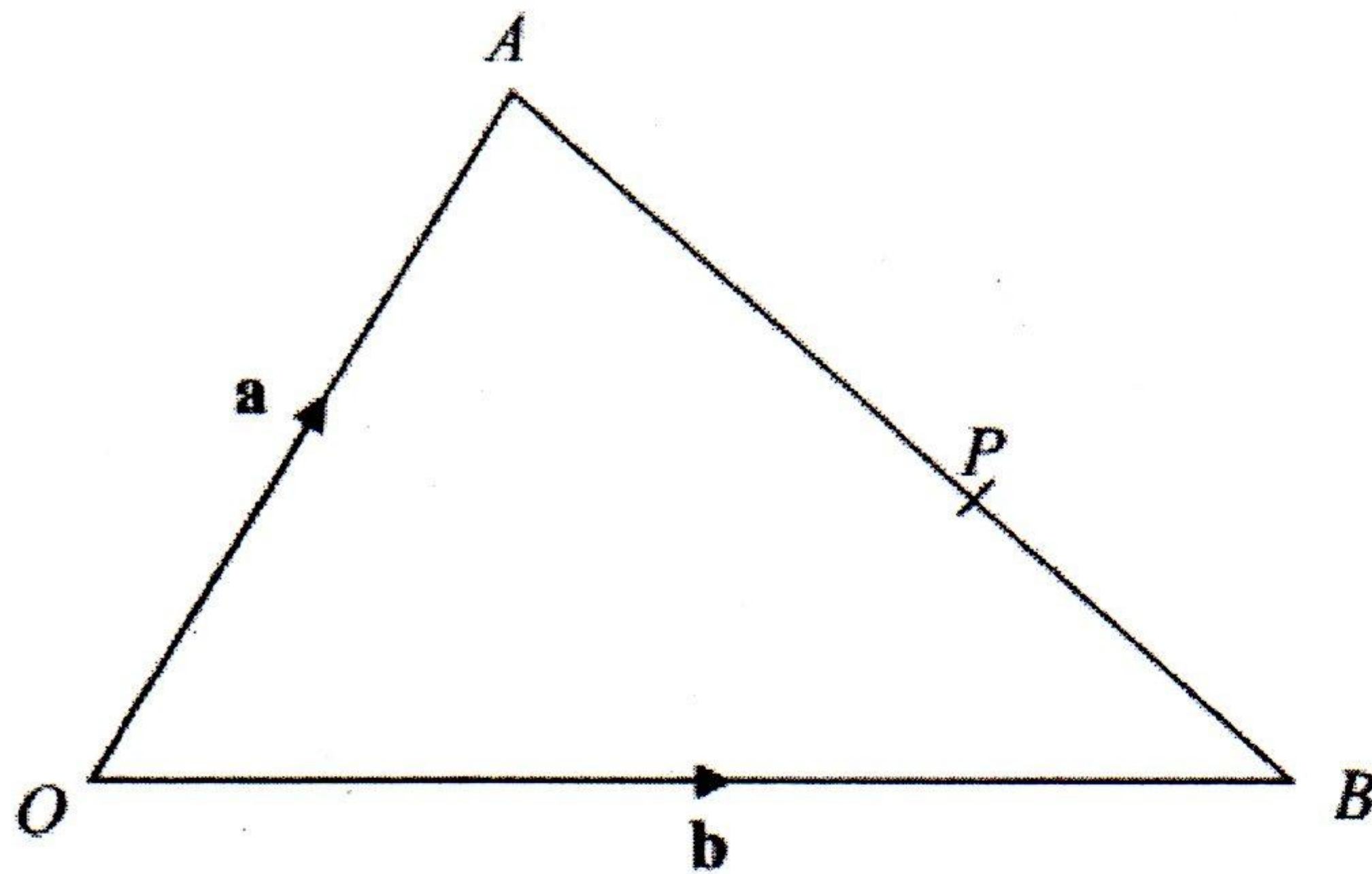
vii $-3(a + b)$

viii $2a + b - 3a - 2b$

ix $2(2a - b) - 3(a - b)$

Vector geometry

Triangular Law of vector



Here in the figure 'O' is the origin, AOB is the triangle

P is the point on AB such that $AP : PB = 3 : 2$

OB vector is given by 'b' and BO vector = -b (Opposite direction)

OA vector given by 'a' and AO vector = -a (Opposite direction)

From the given ratio

$$2AP = 3PB$$

Always express the part of the side as the ration of the full side.

$$AP + PB = AB$$

$$AP = \frac{3}{5} AB \text{ and } PB = \frac{2}{5} AB$$

$$BP + PA = BA$$

$$BP = \frac{2}{5} BA \text{ and } PA = \frac{3}{5} BA$$

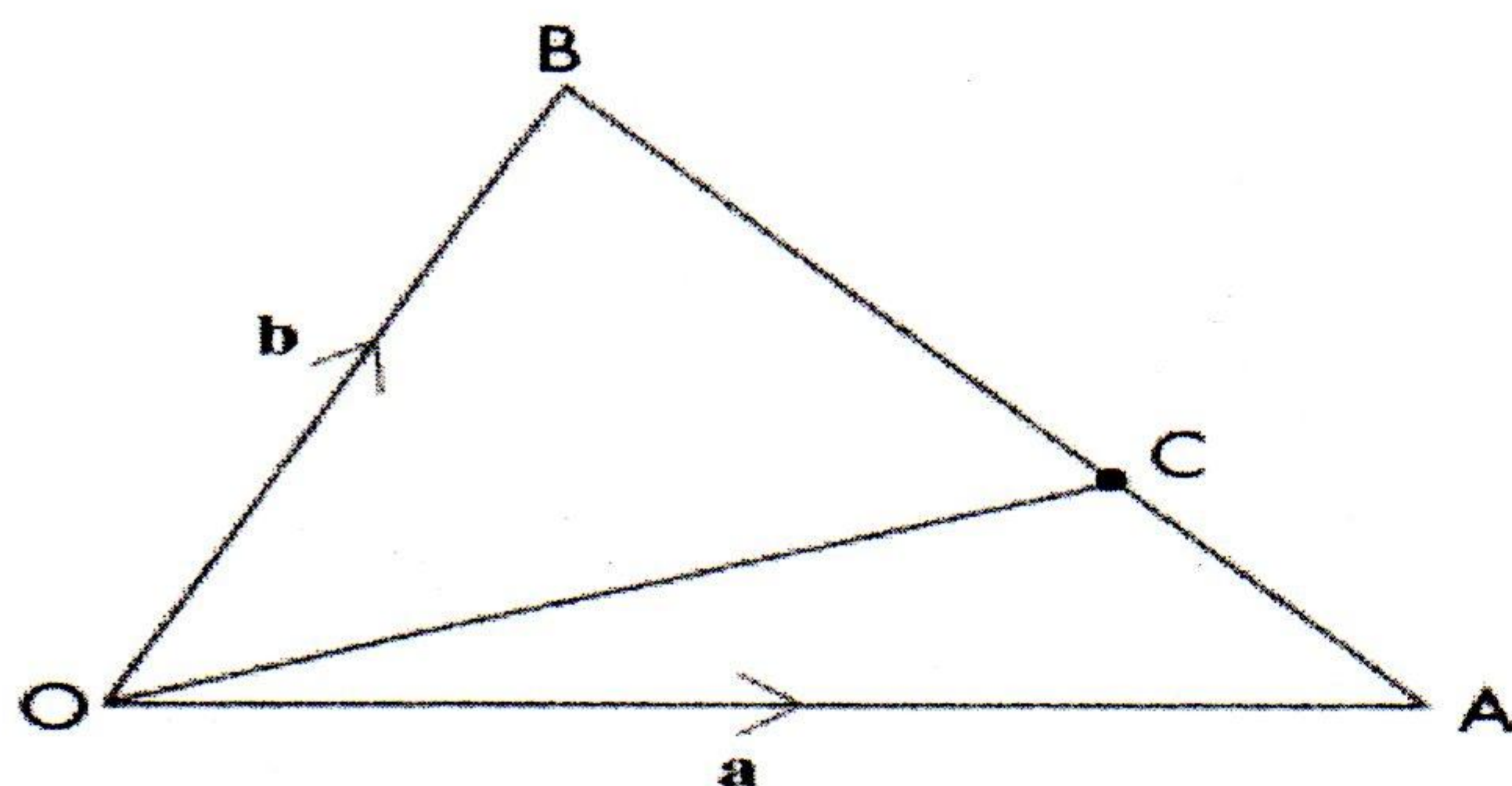
$AB = AO + OB$ $= -a + b$ $OP = OA + AP$ $= a + \frac{3}{5} AB$ $= a + \frac{3}{5} (-a + b)$ $= \frac{5a}{5} + \frac{3}{5} (-a + b)$ $= \frac{1}{5}(5a - 3a + 3b)$ $= \frac{1}{5}(2a + 3b)$	$BA = BO + OA$ $= -b + a$ $OP = OB + BP$ $= b + \frac{2}{5} BA$ $= b + \frac{2}{5} (-b + a)$ $= \frac{5b}{5} + \frac{2}{5} (-b + a)$ $= \frac{1}{5}(5b - 2b + 2a)$ $= \frac{1}{5}(2a + 3b)$
--	--

If P is the midpoint , then the ratio $AP : PB = 1 : 1$

$$AP = \frac{1}{2}(AB)$$

$$\text{Similarly } OP = \frac{1}{2}(a + b)$$

1) The diagram shows the vectors
 $OA = \mathbf{a}$ and $OB = \mathbf{b}$.



The point C divides the line AB in the ratio 1: 3 (i.e. AC is $\frac{1}{4}$ the distance from A to B).

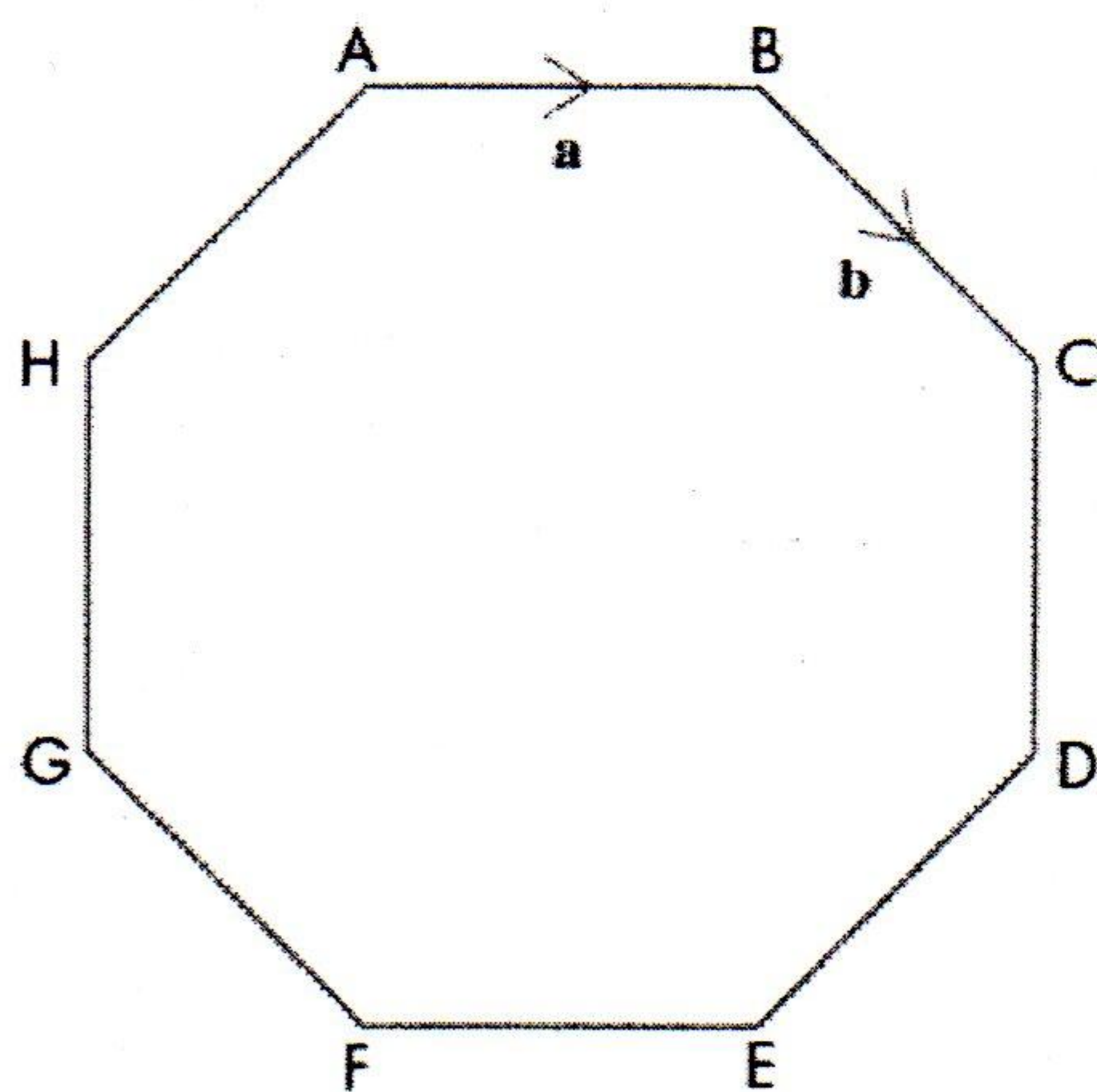
a i Work out the vector AB.

ii Work out the vector AC.

iii Work out the vector OC in terms of \mathbf{a} and \mathbf{b} .

b If C now divides the line AB in the ratio 2: 3 (i.e. AC is $\frac{2}{5}$ the distance from A to B), write down the vector that represents OC.

2) ABCDEFGH is a regular octagon. AB is represented by the vector \mathbf{a} , and BC by the vector \mathbf{b} .



a By means of a diagram, or otherwise, explain why $CD = \sqrt{2}\mathbf{b} - \mathbf{a}$.

b By means of a diagram, or otherwise, explain why $DE = \mathbf{b} - \sqrt{2}\mathbf{a}$.

c Express the following vectors in terms of \mathbf{a} and \mathbf{b} .

i EF

ii FG

iii GH

iv HA

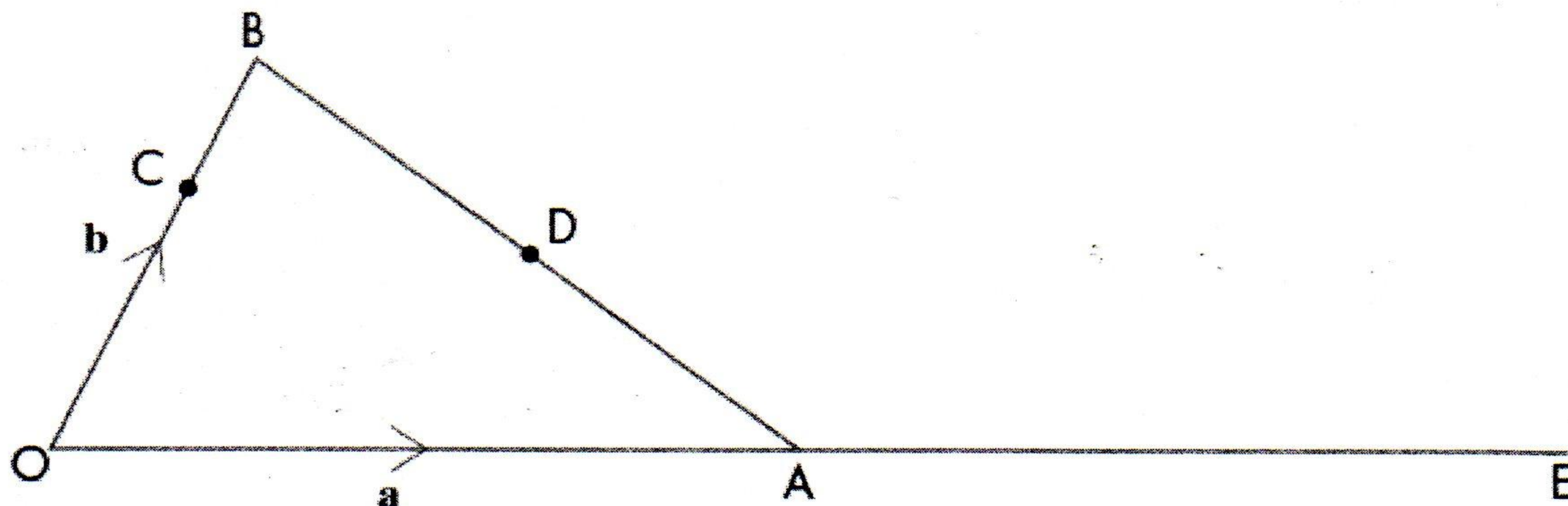
v HC

vi AD

vii BE

viii BF

3) The diagram shows the vectors



$OA = \mathbf{a}$ and $OB = \mathbf{b}$.

The point C divides OB in the ratio 2 : 1 (i.e. OC is $\frac{2}{3}$ the distance from O to B).

The point E is such that $OE = 2 OA$. D is the midpoint of AB.

a Write down (or work out) these vectors in terms of \mathbf{a} and \mathbf{b} .

i OC ii OD iii CO

b The vector CD can be written as $CD = CO + OD$.

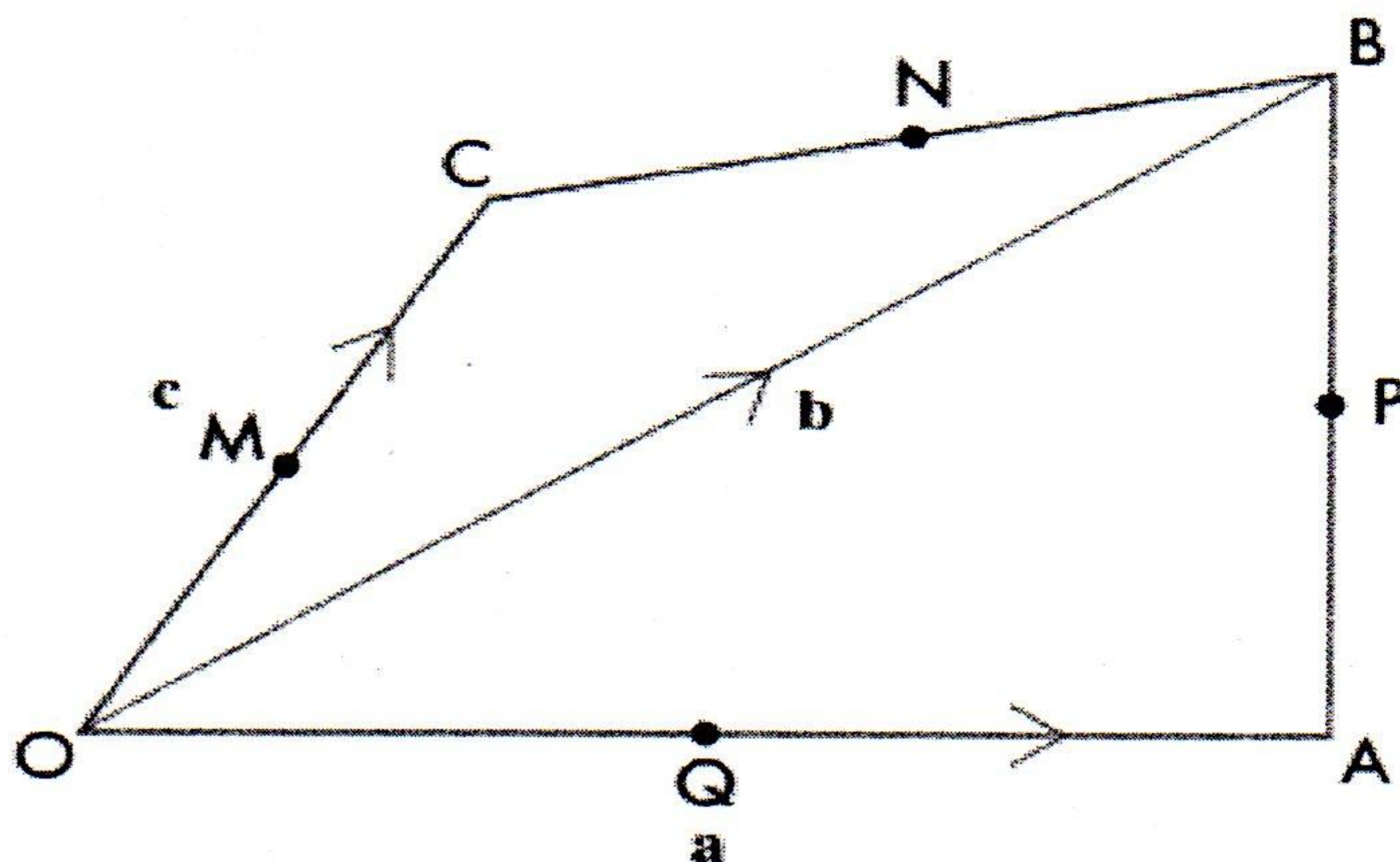
Use this fact to work out CD in terms of \mathbf{a} and \mathbf{b} .

c Write down a similar rule to that in part **b** for the vector DE.

Use this rule to work out DE in terms of \mathbf{a} and \mathbf{b} .

d Explain why C, D and E lie on the same straight line.

4) In the quadrilateral OABC, M, N, P and Q are the midpoints of the sides as shown. OA is represented by the vector \mathbf{a} , and OC by the vector \mathbf{c} . The diagonal OB is represented by the vector \mathbf{b} .



a Express these vectors in terms of \mathbf{a} , \mathbf{b} and \mathbf{c} .

i AB ii AP iii OP

Give your answers as simply as possible.

b i Express the vector ON in terms of \mathbf{b} and \mathbf{c} .

ii Hence express the vector PN in terms of \mathbf{a} and \mathbf{c} .

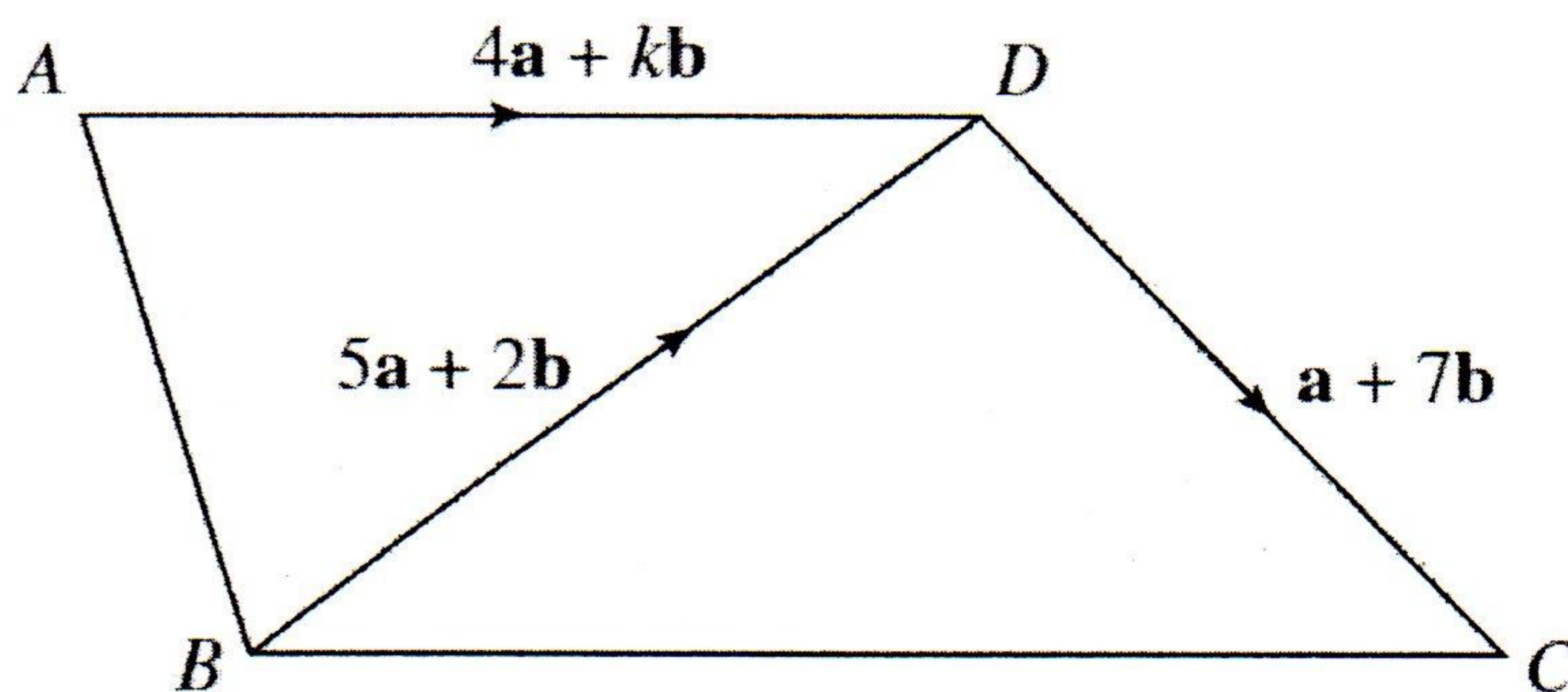
c i Express the vector QM in terms of \mathbf{a} and \mathbf{c} .

ii What relationship is there between PN and QM?

iii What sort of quadrilateral is PNMQ?

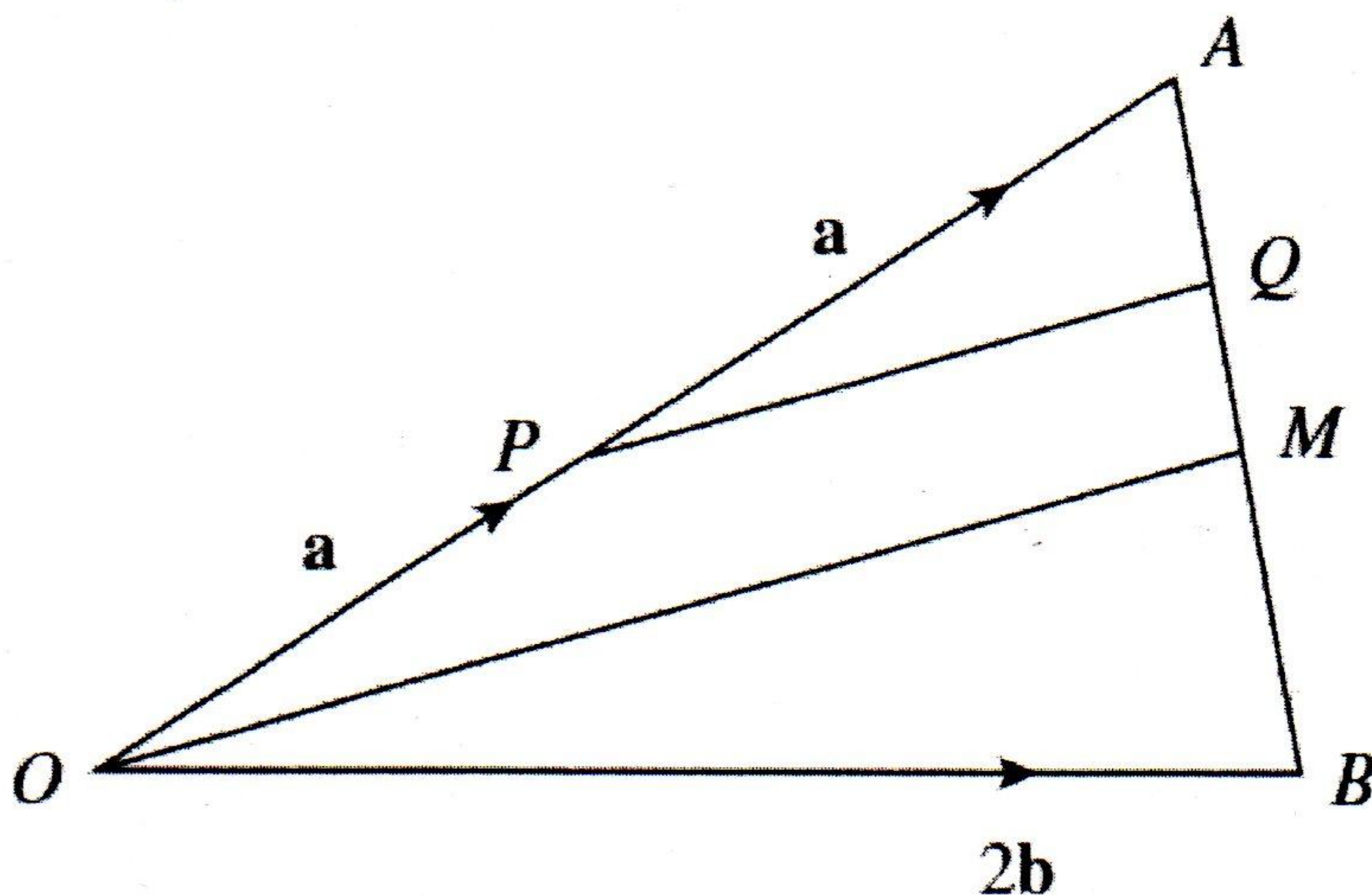
d Prove that $AC = 2 QM$.

1) $ABCD$ is a trapezium. BC is parallel to AD .
 $BD = 5\mathbf{a} + 2\mathbf{b}$, $DC = \mathbf{a} + 7\mathbf{b}$ and $AD = 4\mathbf{a} + k\mathbf{b}$, where k is a number to be determined.



Find the value of k .
 You **must** show your working.

2) OAB is a triangle with P the mid-point of OA and M the mid-point of AB .
 $OP = \mathbf{a}$, $PA = \mathbf{a}$ and $OB = 2\mathbf{b}$



(a) Write down an expression for AB in terms of \mathbf{a} and \mathbf{b} .

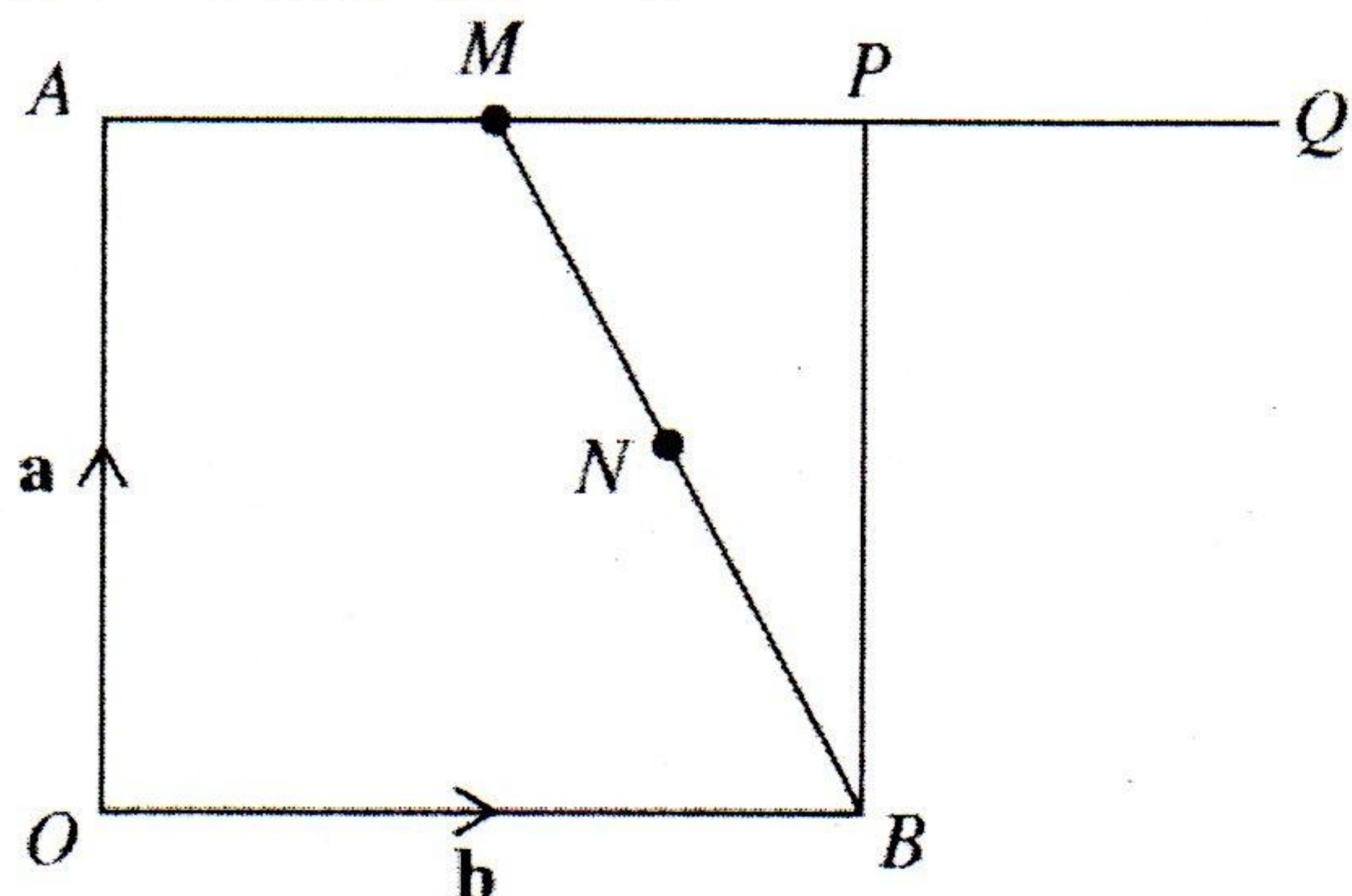
Q lies on AB such that $AQ = \frac{1}{4} AB$
 Show that $PQ = \frac{1}{2} \mathbf{a} + \frac{1}{2} \mathbf{b}$

Explain your answer.

(c) Write down, and simplify, an expression for OM in terms of \mathbf{a} and \mathbf{b} .

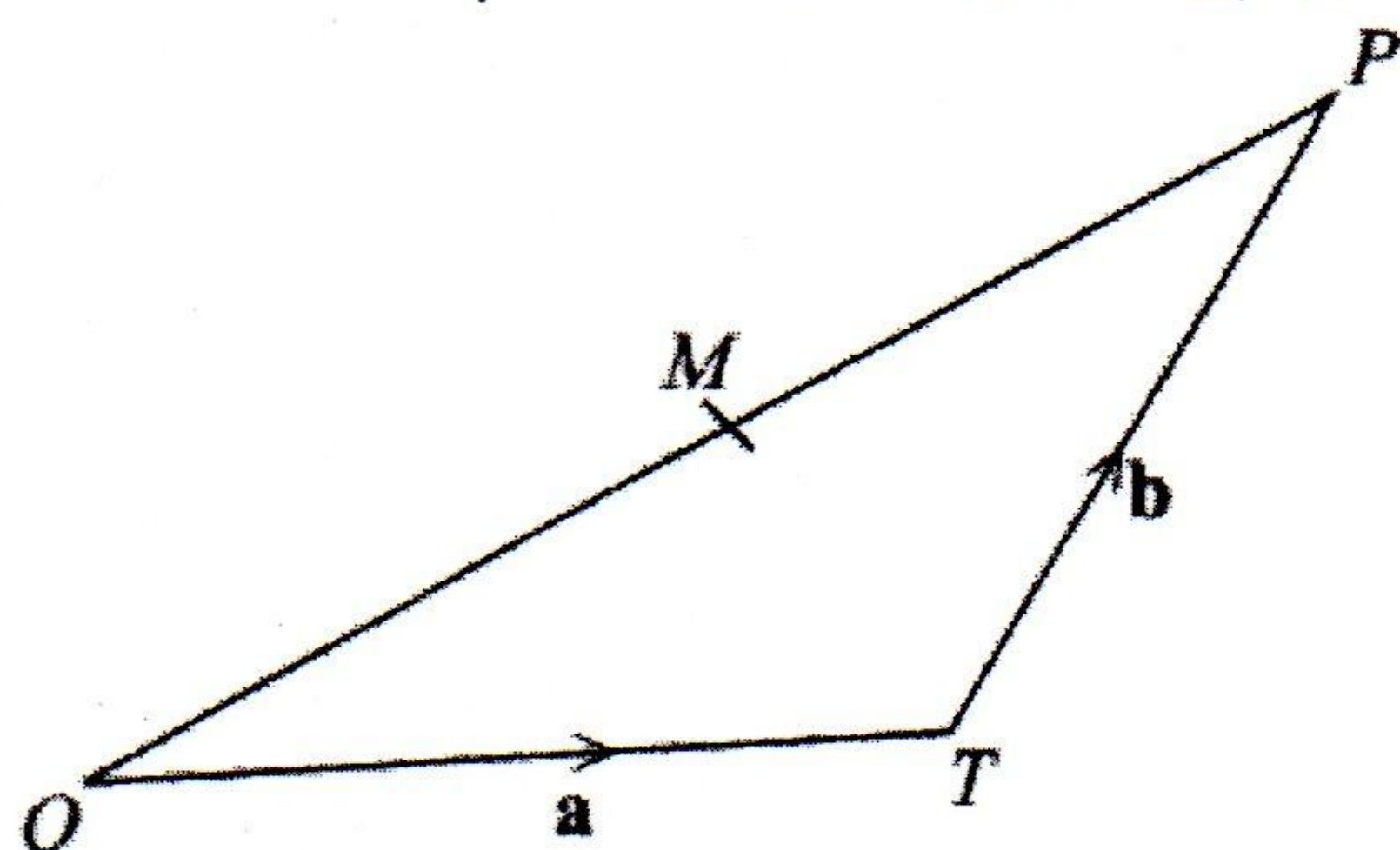
(d) Explain why the answers for part (b) and part (c) show that $OPQM$ is a trapezium.

- 3) The diagram shows a square $OAPB$.
 M is the mid-point of AP . N is the mid-point of BM .
 AP is extended to Q where $AQ = 2 AP$
 $OA = \mathbf{a}$ and $OB = \mathbf{b}$



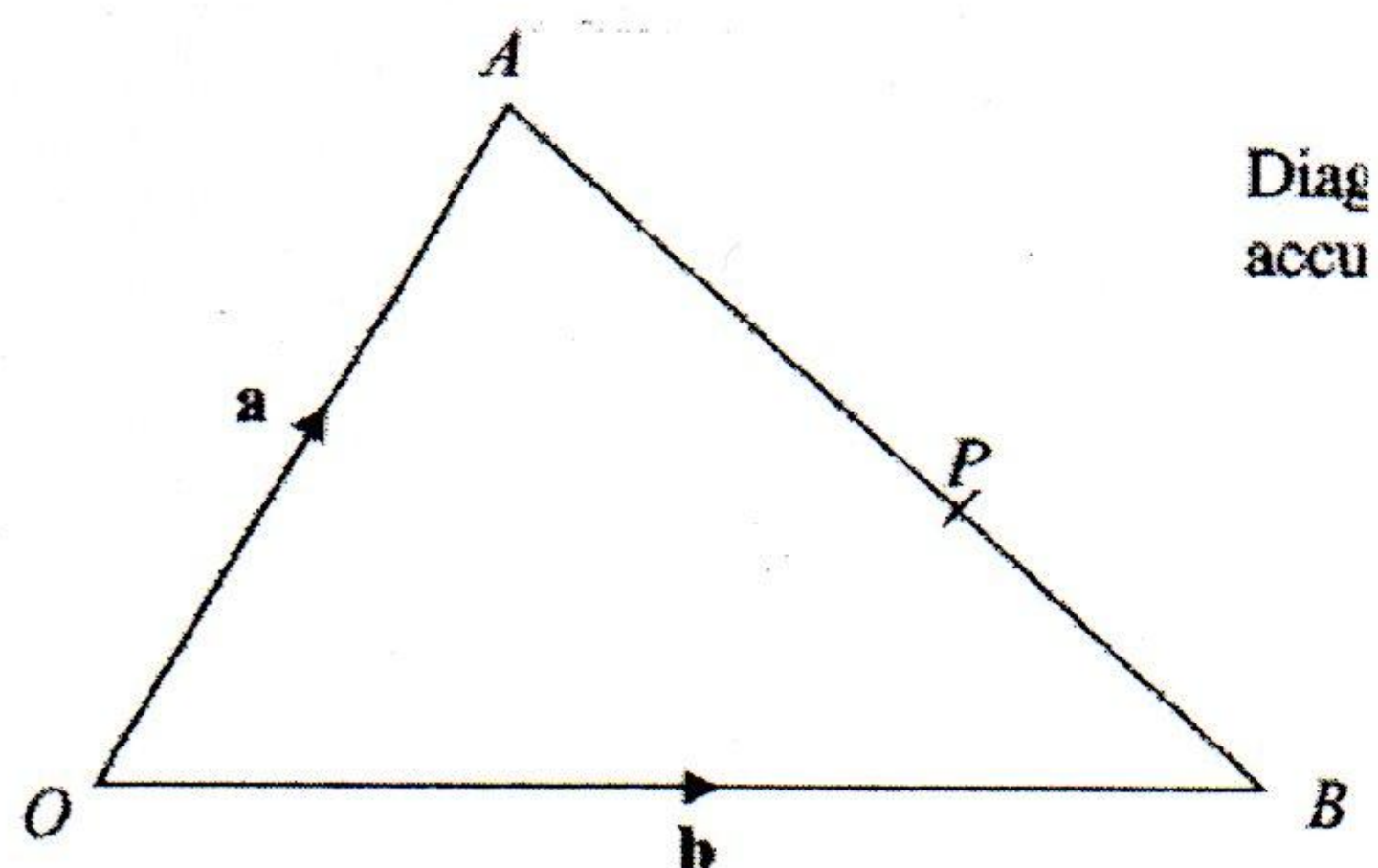
- a) Write these vectors in terms of \mathbf{a} and \mathbf{b} . Give your answers in their simplest form.
 (i) OQ (ii) BM (iii) BN (iv) ON
- (b) What can you deduce about points O , N and Q ?
 Give a reason for your answer.

- 4) OPT is a triangle.
 M is the midpoint of OP . $OT = \mathbf{a}$, $TP = \mathbf{b}$



- (a) Express OM in terms of \mathbf{a} and \mathbf{b} . $OM = \dots\dots\dots$
- (b) Express TM in terms of \mathbf{a} and \mathbf{b} .
 Give your answer in its simplest form. $TM = \dots\dots\dots$

- 5) Here OAB is a triangle. $OA = \mathbf{a}$ $OB = \mathbf{b}$

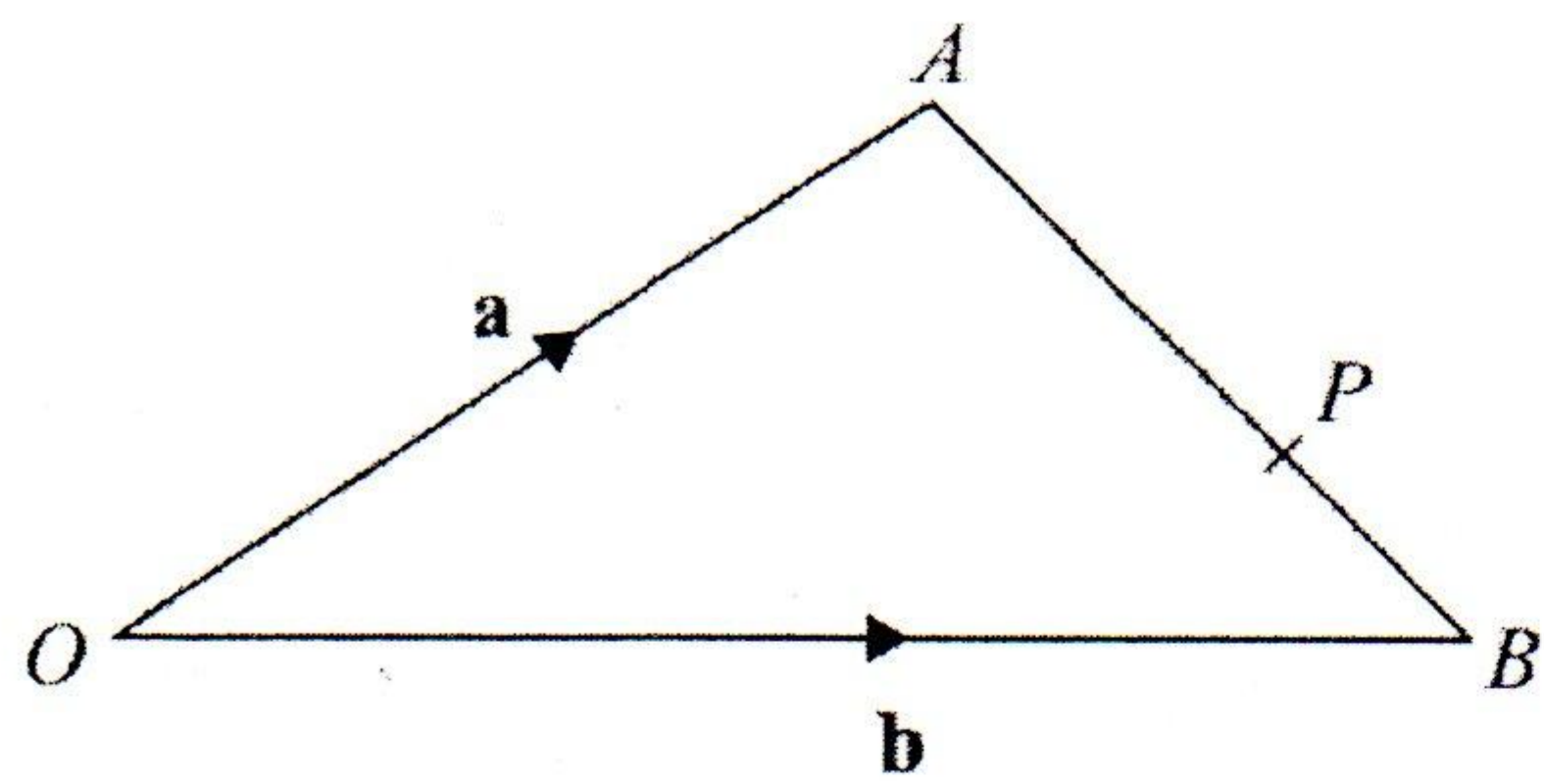


- (a) Find the vector AB in terms of \mathbf{a} and \mathbf{b} . $AB = \dots\dots\dots$

P is the point on AB such that $AP : PB = 3 : 2$

- (b) Show that $OP = \frac{1}{5} (2\mathbf{a} + 3\mathbf{b})$

6) OAB is a triangle. $OA = \mathbf{a}$, $OB = \mathbf{b}$

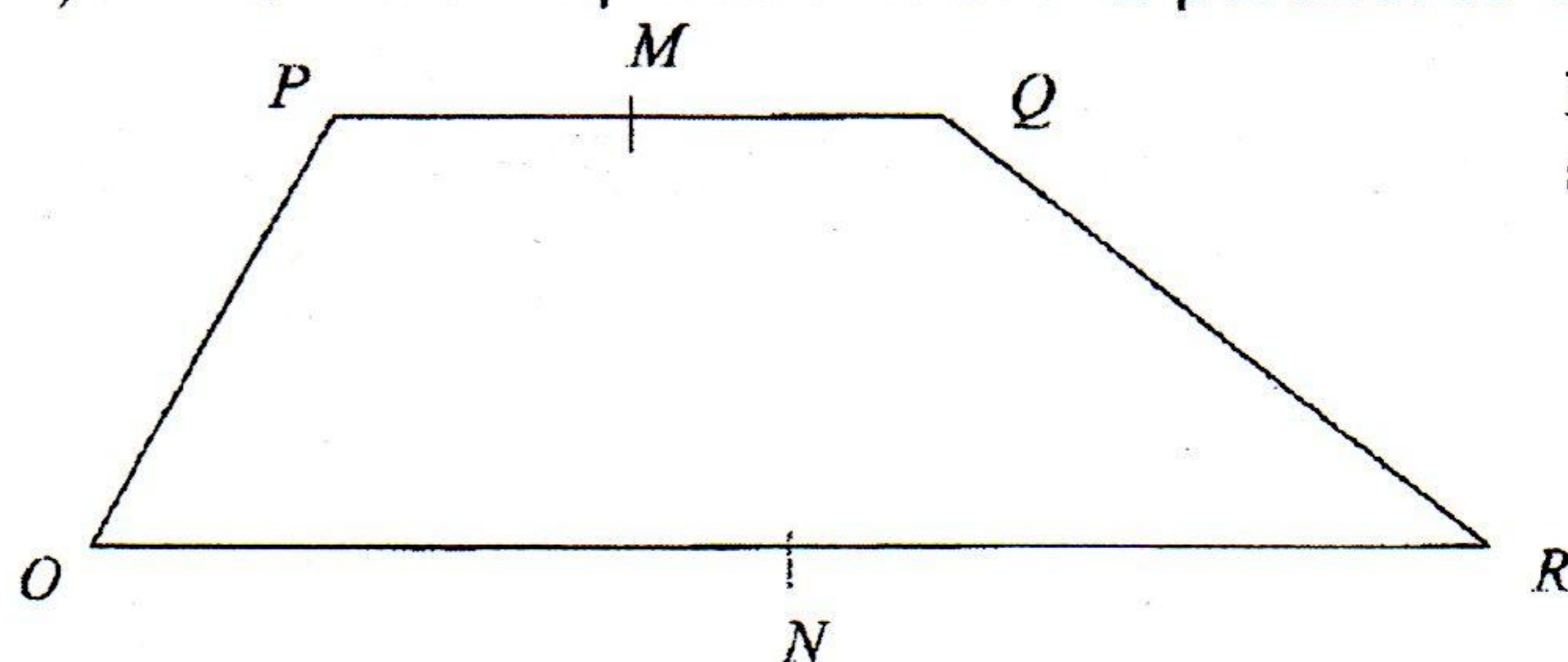


(a) Find the vector AB in terms of \mathbf{a} and \mathbf{b} . $AB = \dots\dots\dots$

P is the point on AB so that $AP : PB = 2 : 1$

(b) Find the vector OP in terms of \mathbf{a} and \mathbf{b} .
Give your answer in its simplest form. $OP = \dots\dots\dots$

7) $OPQR$ is a trapezium with PQ parallel to OR



$OP = 2\mathbf{b}$ $PQ = 2\mathbf{a}$ $OR = 6\mathbf{a}$

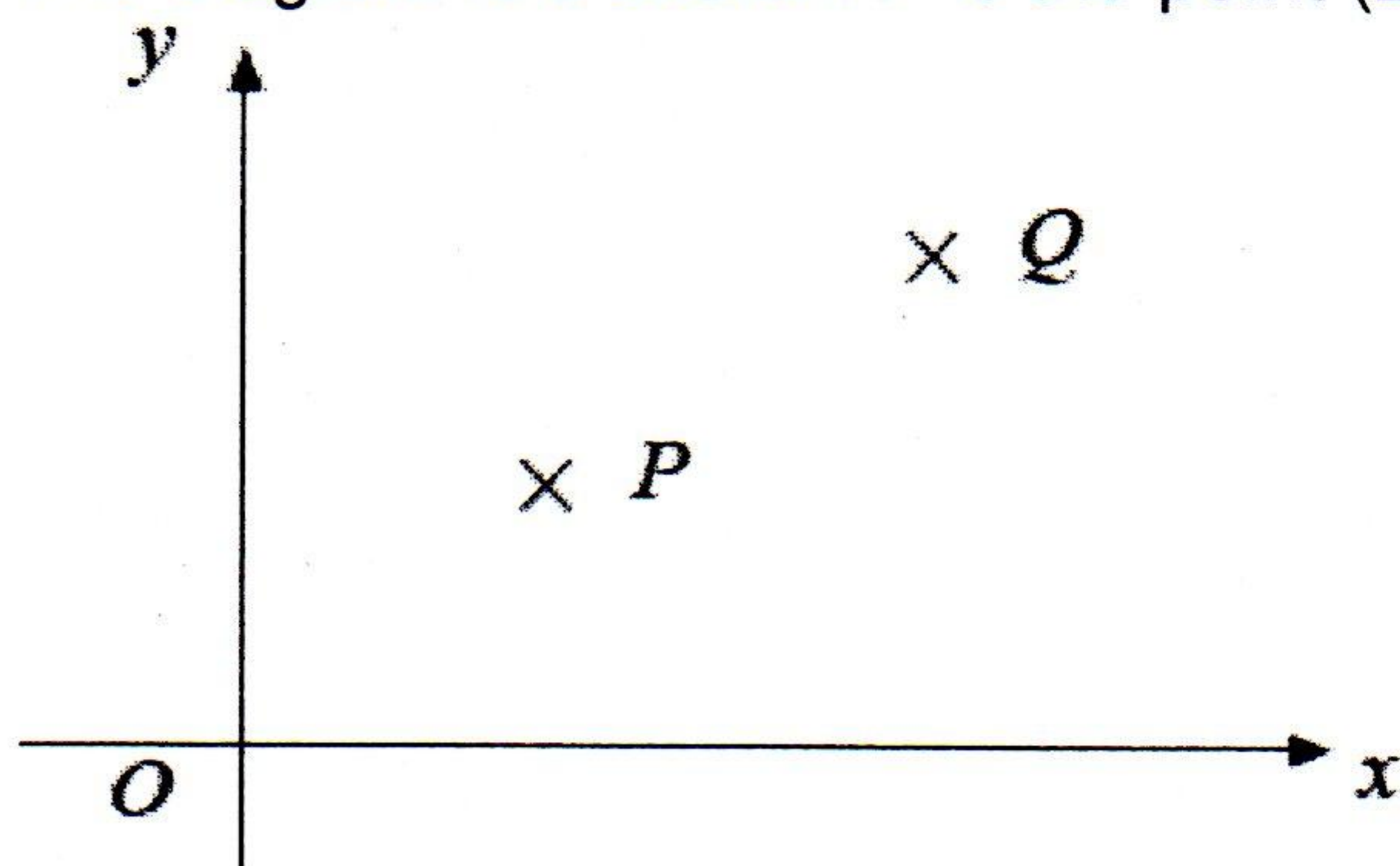
M is the midpoint of PQ and N is the midpoint of OR .

a) Find the vector MN in terms of \mathbf{a} and \mathbf{b} .

X is the midpoint of MN and Y is the midpoint of QR .

b) Prove that XY is parallel to OR .

8) The diagram is a sketch. P is the point $(2, 3)$ Q is the point $(6, 6)$



The diagram is a sketch. P is the point $(2, 3)$ Q is the point $(6, 6)$

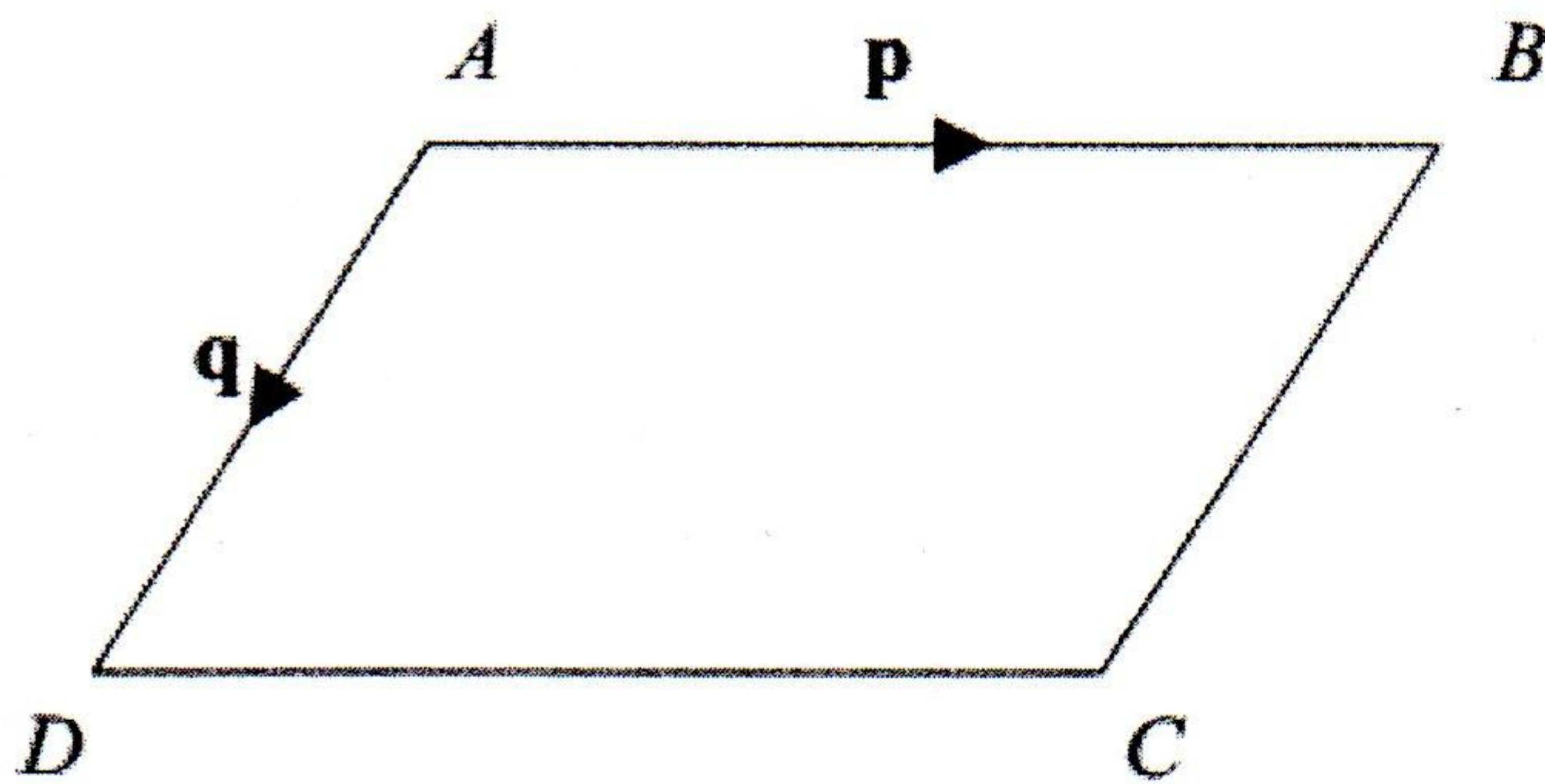
a) Write down the vector PQ .

Write your answer as a column vector.

$PQRS$ is a parallelogram. And $PR = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$

Find the vector QS

9) $ABCD$ is a parallelogram. AB is parallel to DC . AD is parallel to BC .



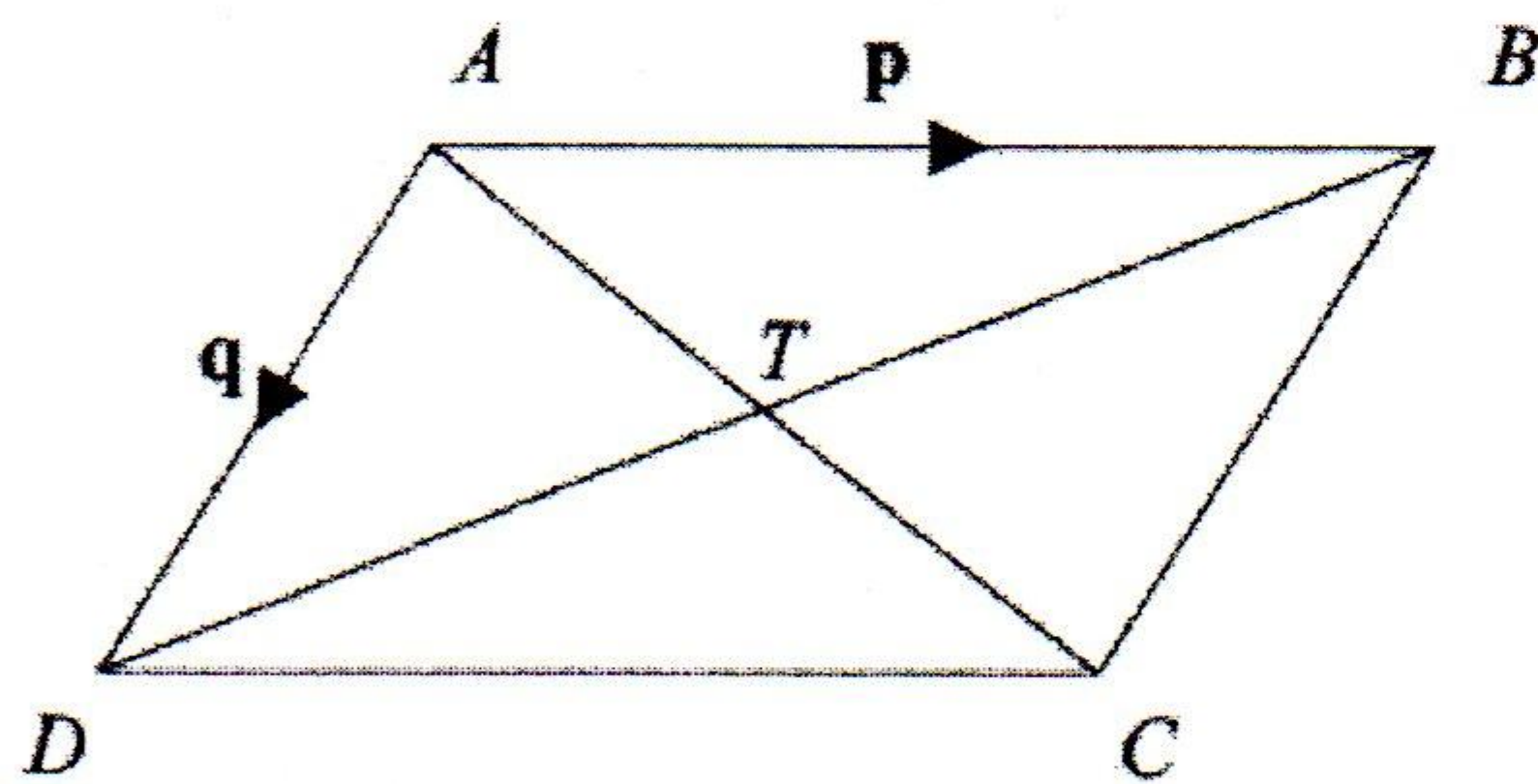
\overrightarrow{AB} Vector = \mathbf{p} and \overrightarrow{AD} vector = \mathbf{q} .

a) Express following vectors, in terms of \mathbf{p} and \mathbf{q}

i) \overrightarrow{AC}

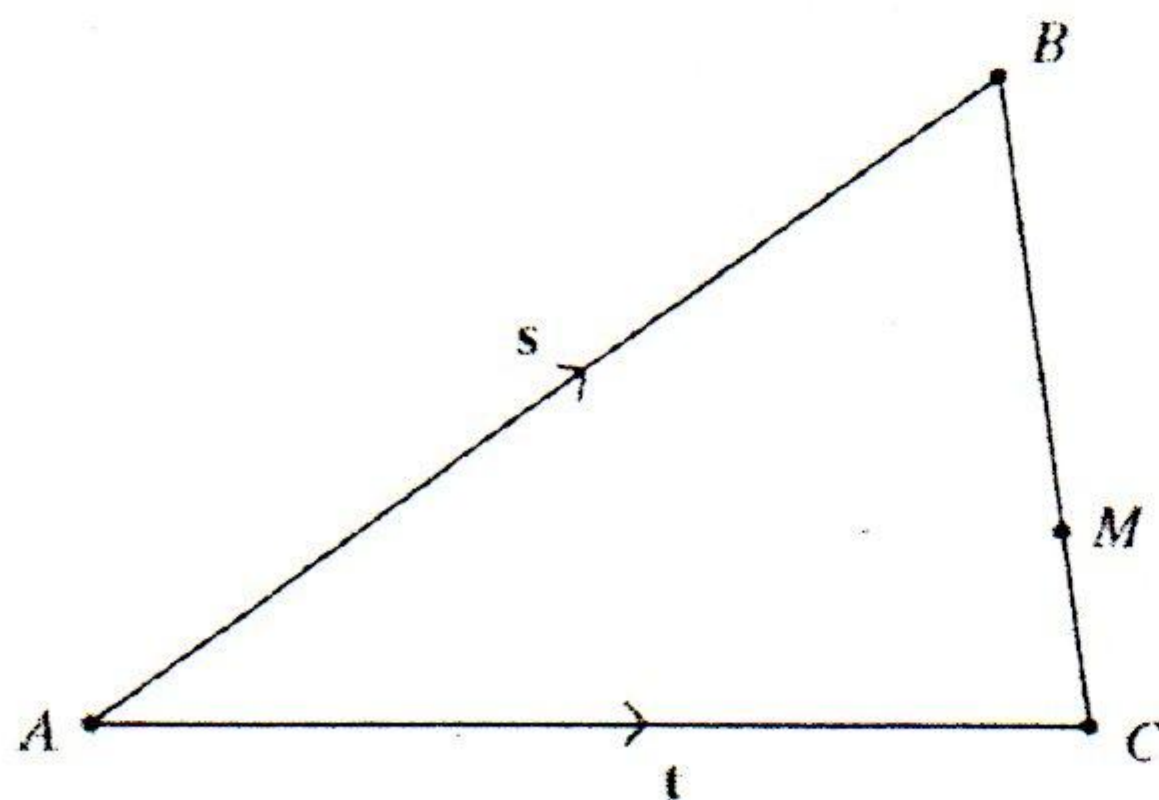
ii) \overrightarrow{BD}

10) AC and BD are diagonals of parallelogram $ABCD$
 AC and BD intersect at T



Express \overrightarrow{AT} in terms of \mathbf{p} and \mathbf{q} .

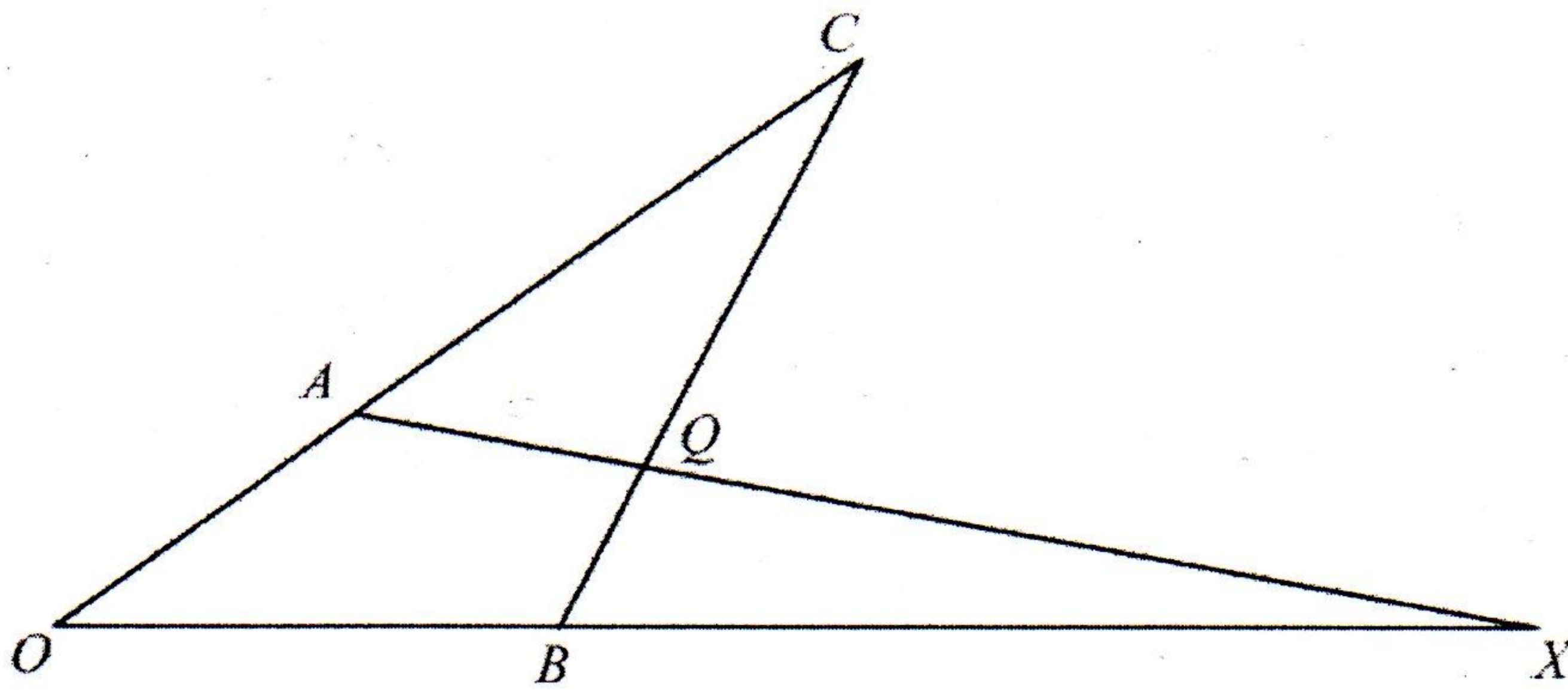
11) In triangle ABC , M lies on BC such that $BM = \frac{3}{4} BC$
 $\overrightarrow{AB} = \mathbf{s}$ and $\overrightarrow{AC} = \mathbf{t}$



Find \overrightarrow{AM} in terms of \mathbf{s} and \mathbf{t} .

Give your answer in its simplest form

12) In the diagram, $OA=4a$ and $OB=4$



OAC , OBX and BQC are all straight lines
 $AC = 2OA$ and $BQ:QC = 1:3$

(a) Find, in terms of **a** and **b**, the vectors which represent

(i) BC

(ii) AQ

Given that $BX = 8b$

(b) Show that AQX is a straight line.